



# **Timed and Hybrid Systems**

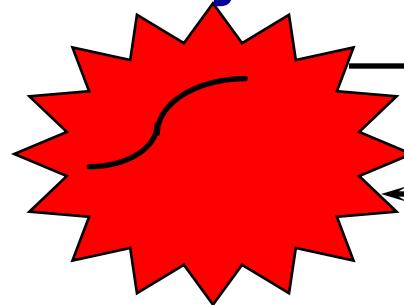
*in UPPAAL2k*

**Kim Guldstrand Larsen**  
**BRICS@Aalborg**

**Paul Pettersson**  
**BRICS@Aalborg & DoCS@Uppsala**

# Hybrid & Real Time Systems

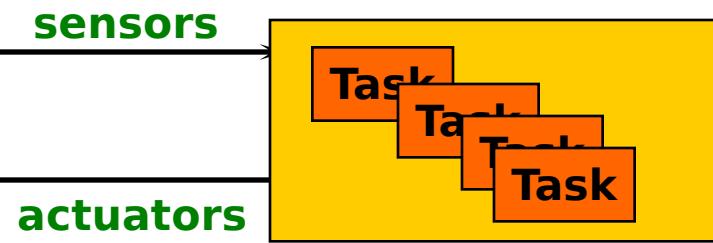
## Control Theory



### Plant

*Continuous*

## Computer Science



### Controller Program

*Discrete*

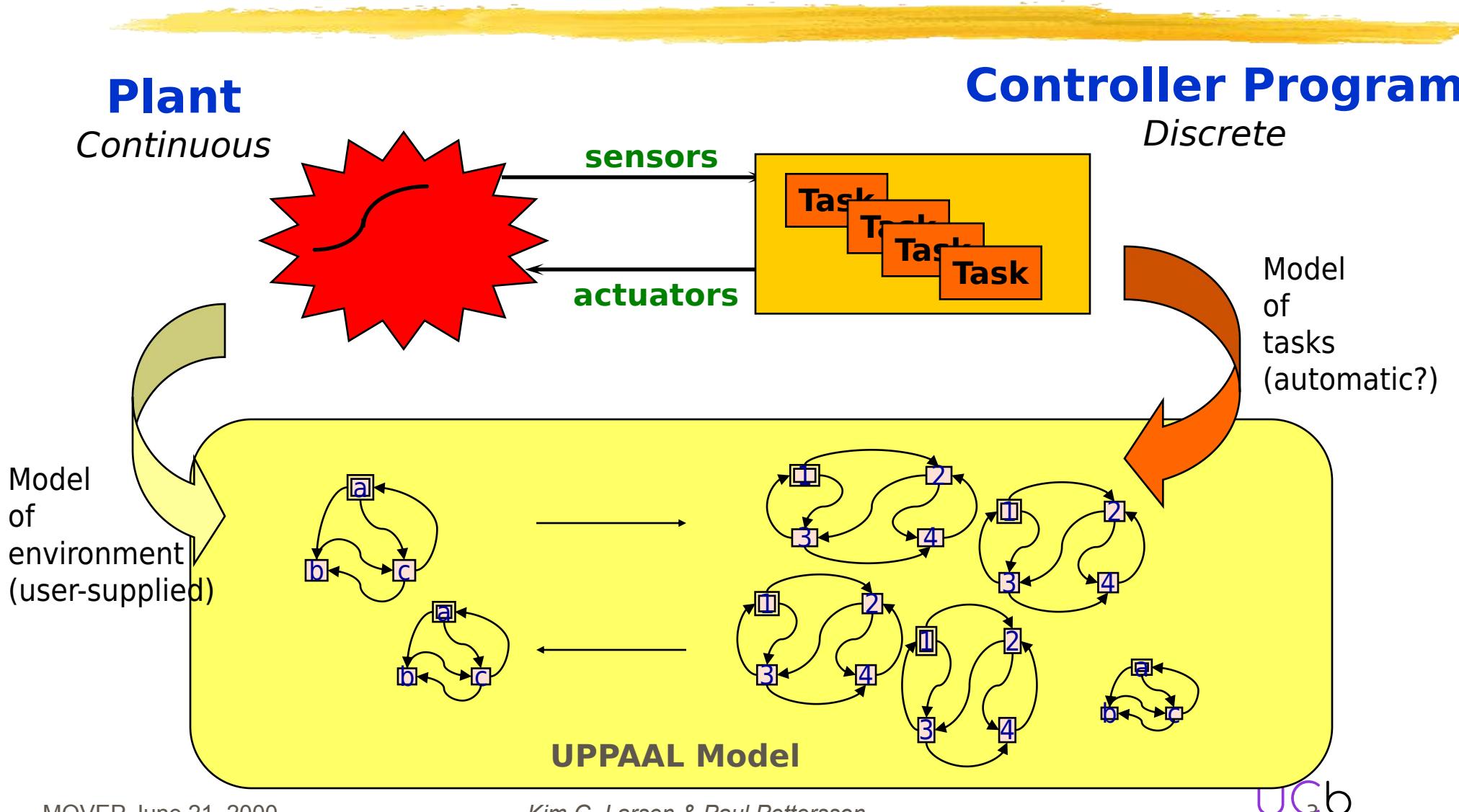
- Eg.:**
- Pump Control
  - Air Bags
  - Robots
  - Cruise Control
  - ABS
  - CD Players
  - Production Lines

### Real Time System

A system where correctness not only depends on the logical order of events but also on their **timing!!**

# Validation & Verification

## *Construction of UPPAAL models*

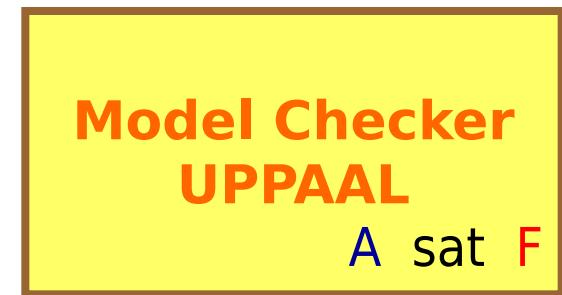


# Real Timed Model Checking

History

System Description  
Timed Automata **A**

Requirement  
Specification **F**



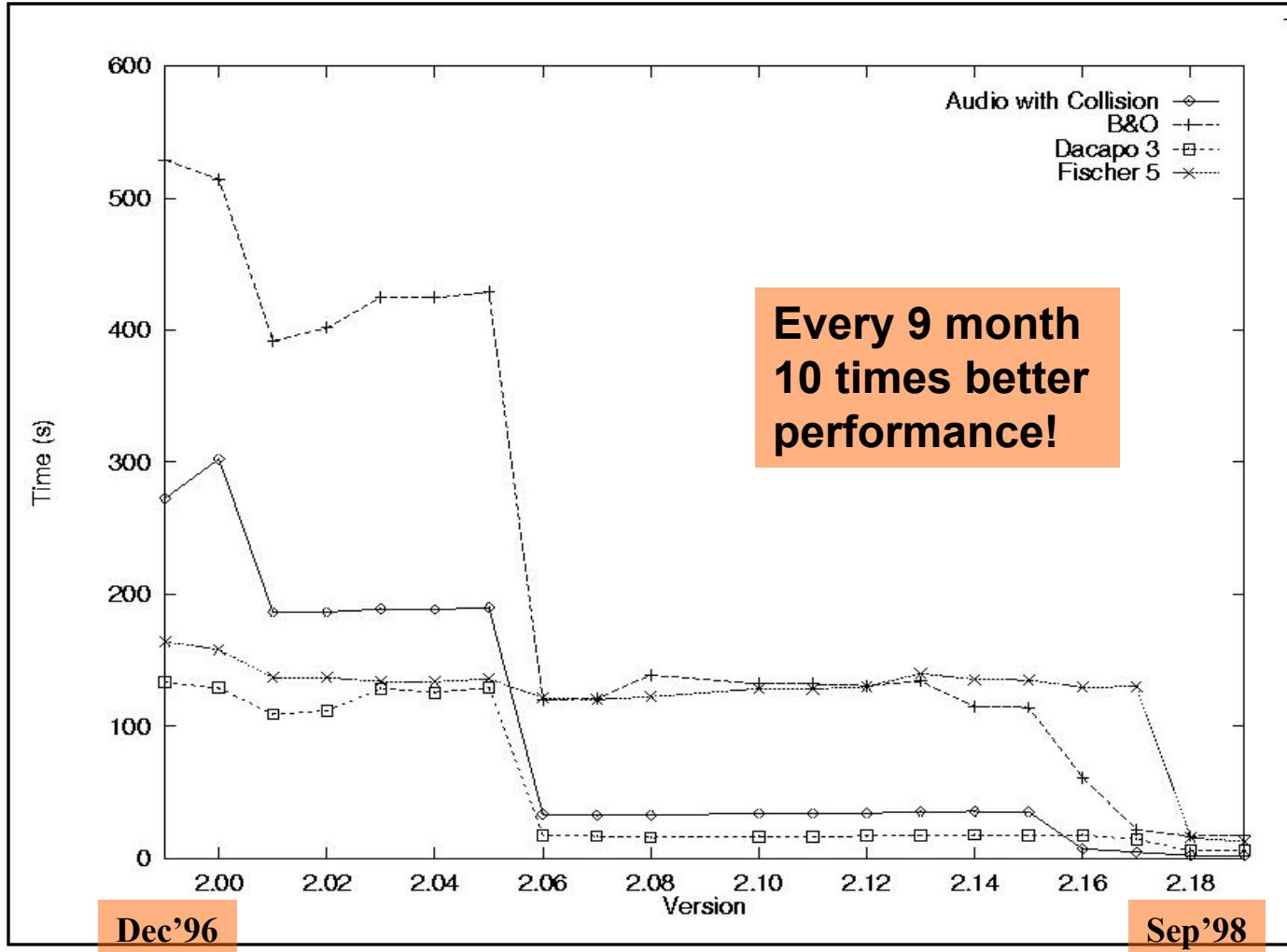
Yes!

No!

Diagnostic Information



# UPPAAL 1995 - 2000



with  
Validator

L 2.17

# Collaborators

## ✉ @UPPsala

- ⊖ Wang Yi
- ⊖ Johan Bengtsson
- ⊖ Paul Pettersson
- ⊖ Fredrik Larsson
- ⊖ Alexandre David
- ⊖ Tobias Amnell

## ✉ @AALborg

- ⊖ Kim G Larsen
- ⊖ Arne Skou
- ⊖ Paul Pettersson
- ⊖ Carsten Weise
- ⊖ Kåre J Kristoffersen
- ⊖ Gerd Behrman
- ⊖ Thomas Hune

## ✉ @Elsewhere

- ⊖ Franck Cassez, Magnus Lindahl, Francois Laroussinie, Patricia Bouyer, Augusto Burgueno, David Griffioen, Ansgar Fehnker, Frits Vandraager, Klaus Havelund, Theo Ruys, Pedro D'Argenio, J-P Katoen, J. Tretmans, H. Bowmann, D. Latella, M. Massink, G. Faconti, Kristina Lundqvist, Lars Asplund, Justin Pearson...

# Overview of Tutorial

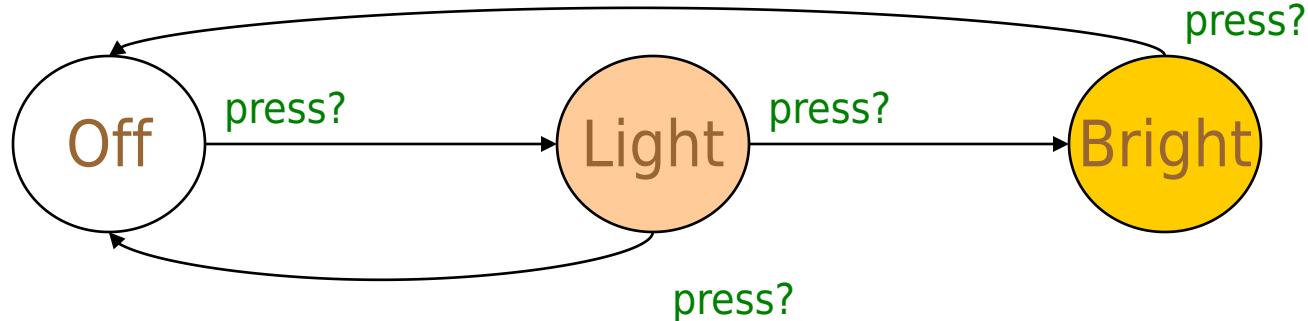
- 🕒 Timed Automata
- 🕒 UPPAAL and LEGO MINDSTORM
- 🕒 The UPPAAL Engine
  - ⊖ Symbolic Reachability
  - ⊖ Verification Options
- 🕒 Applications/Case Studies
  - ⊖ SIDMAR Steel Production Plant (VHS project)



Alur, Dill 1990

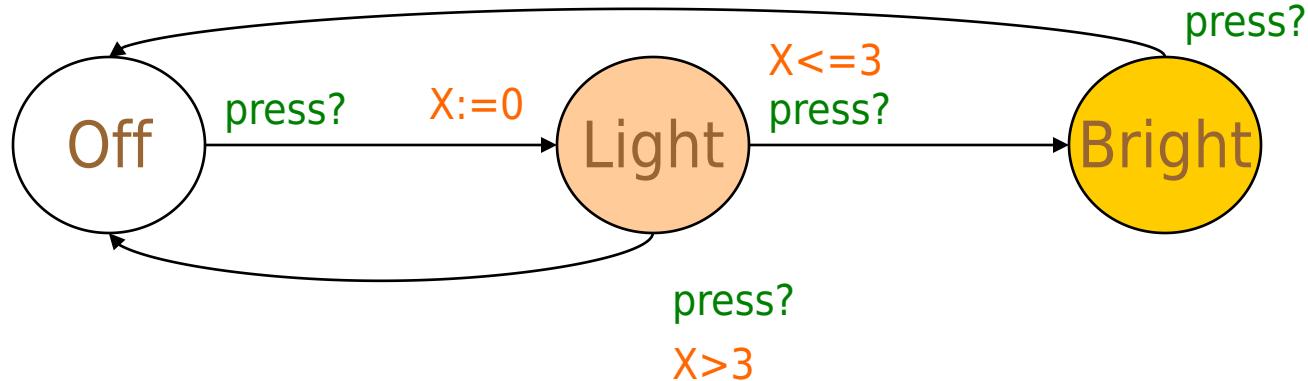
# TIMED AUTOMATA

# Intelligent Light Control



**WANT:** if **press** is issued twice **quickly**  
then the **light** will get **brighter**; otherwise the light is  
turned **off**.

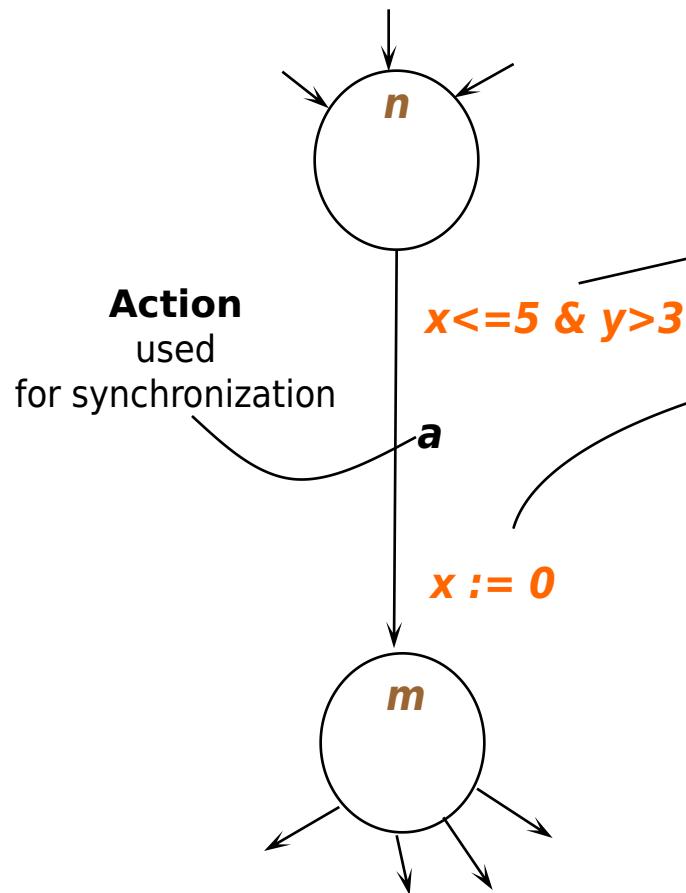
# Intelligent Light Control



**Solution:** Add real-valued clock  $x$

# Timed Automata

Alur & Dill 1990



**Clocks:**  $x, y$

**Guard**  
Boolean combination of integer bounds on clocks and clock-differences.

**Reset**  
Action performed on clocks

**State**

( *location* ,  $x=v$  ,  $y=u$  ) where  $v,u$  are in  $\mathbb{R}$

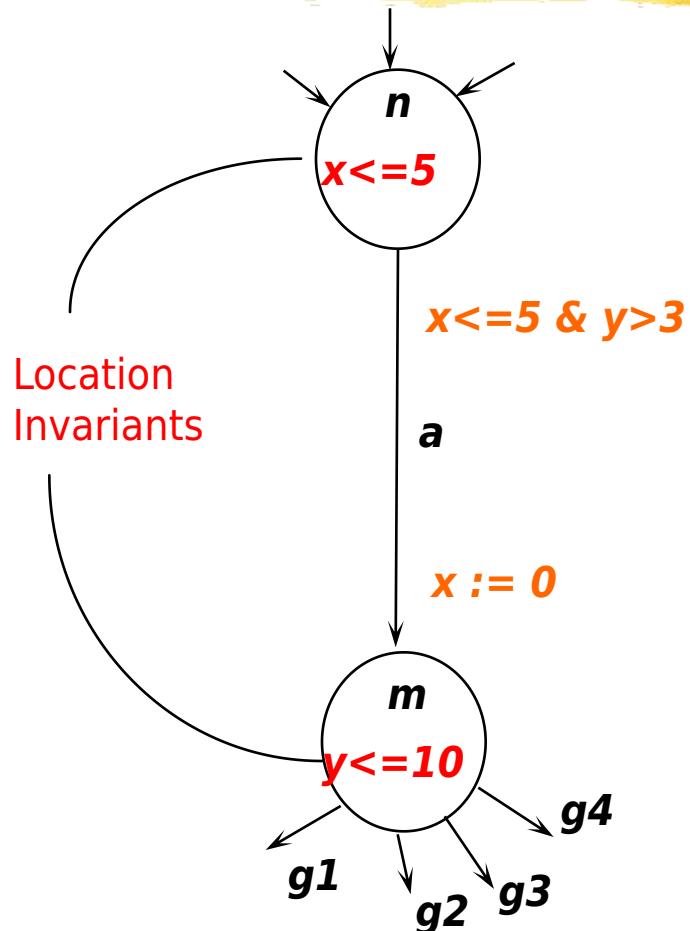
**Transitions**

$$( n , x=2.4 , y=3.1415 ) \xrightarrow{a} ( m , x=0 , y=3.1415 )$$

$$( n , x=2.4 , y=3.1415 ) \xrightarrow{e(1.1)} ( n , x=3.5 , y=4.2415 )$$

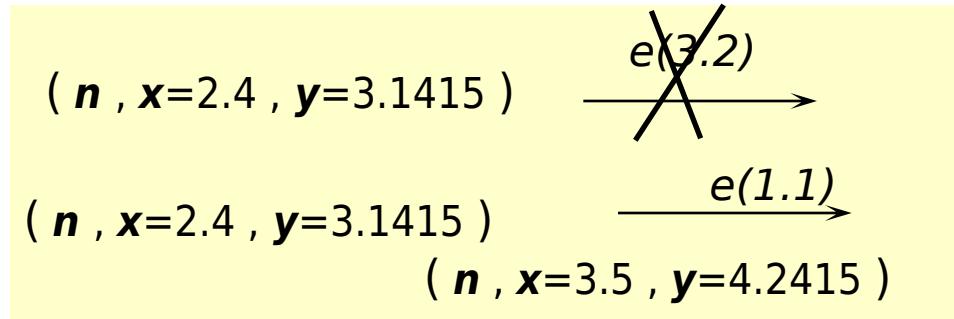
# Timed Automata -

## Invariants



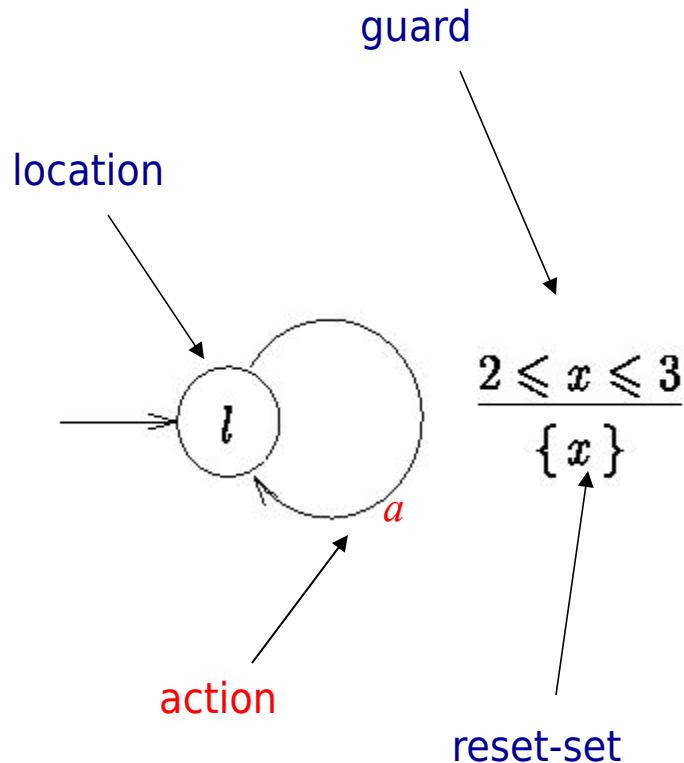
Clocks:  $x, y$

Transitions

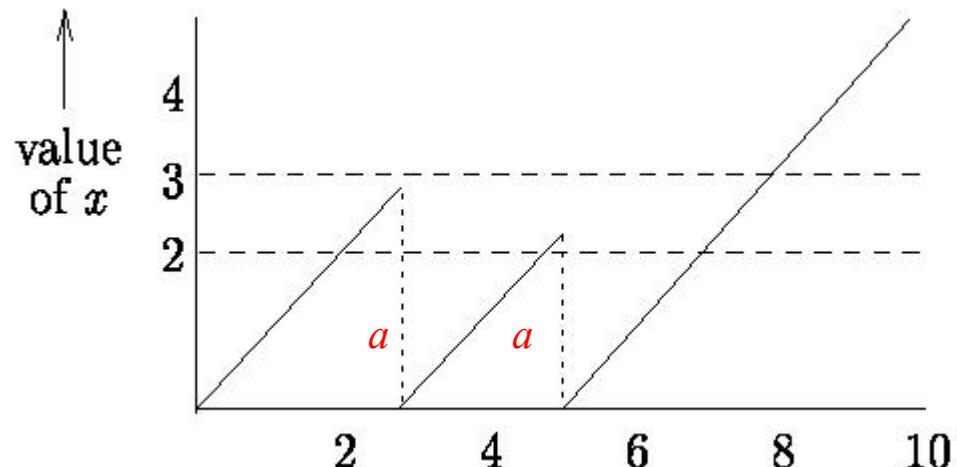
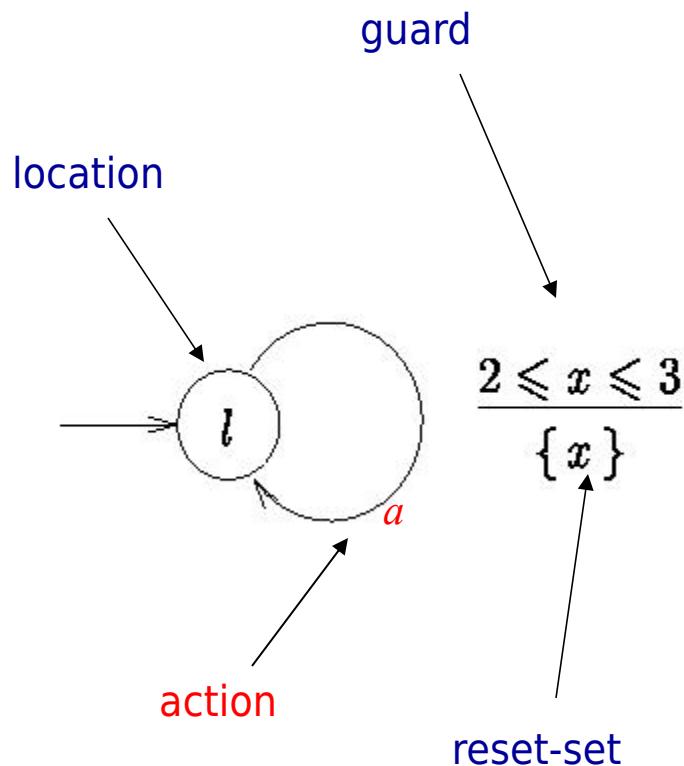


**Invariants insure progress!!**

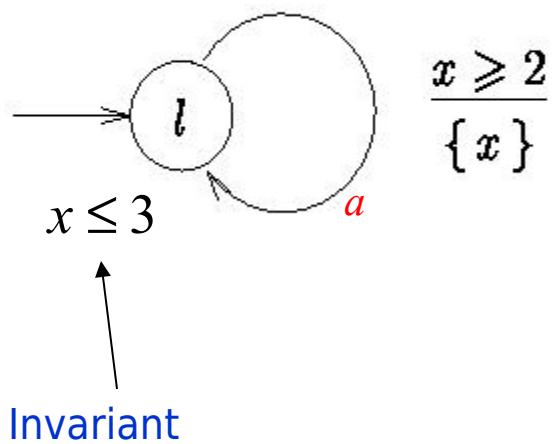
# Timed Automata: Example



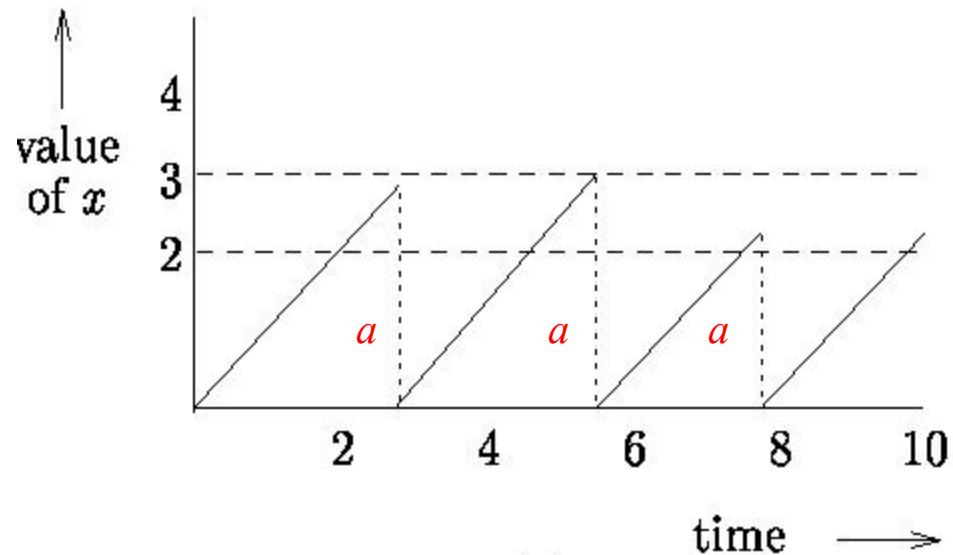
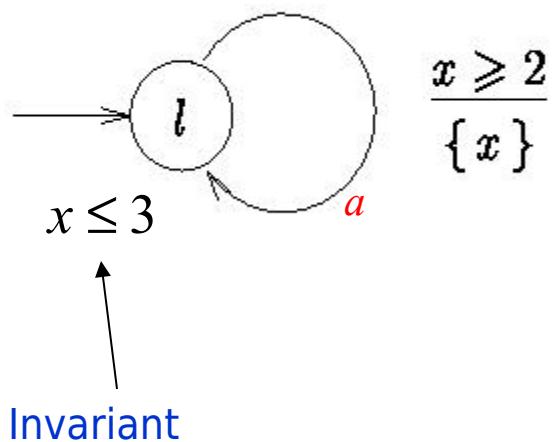
# Timed Automata: Example



# Timed Automata: Example



# Timed Automata: Example



# Fundamental Results

PSPACE-C

Reachability      Alur, Dill

Trace-inclusion      Alur, Dill

$\ominus$  Timed ; Untimed

Bisimulation

$\ominus$  Timed      Cerans ; Untimed

Model-checking

$\ominus$  TCTL, T<sub>mu</sub>, L<sub>nu</sub>, ...

PSPACE-C / EXPTIME-C

# Updatable Timed Automata

	Diagonal-free	General
$x := c, x := y$	Pspace complete	Pspace complete
$x := x + 1$		Undecidable
$x := y + c$		
$x := x - 1$		Undecidable
$x :< c, x :\leq c$	Pspace complete	Pspace complete
$x :> c, x :\geq c$		Undecidable
$x : \sim y + c$		
$(y+)c <: x :< (y+)d$		
$y + c <: x :< z + d$		Undecidable

With  $\sim \in \{<, \leq, \geq, >\}$  and  $c, d \in \mathbb{Q}_+$

Patricia Bouyer, Catherine Dufourd,  
Emmanuel Fleury, Antoine Petit

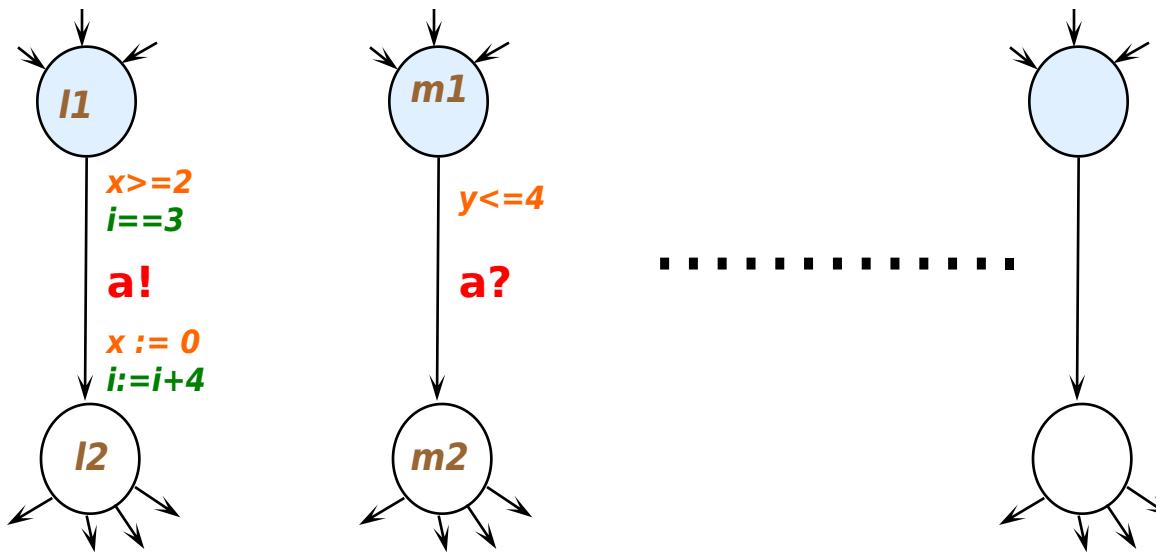
	Diagonal-free	General
$:= c, x := y$	TA-bisimilar	TA-bisimilar
$x := x + 1$		Turing
$x := y + c$		
$:< c, x :\leq c$		TA $_{\epsilon}$
$x :> c, x :\geq c$	TA $_{\epsilon}$	
$x : \sim y + c$		Turing
$(y+)c <: x :< (y+)d$		

# Other Extensions

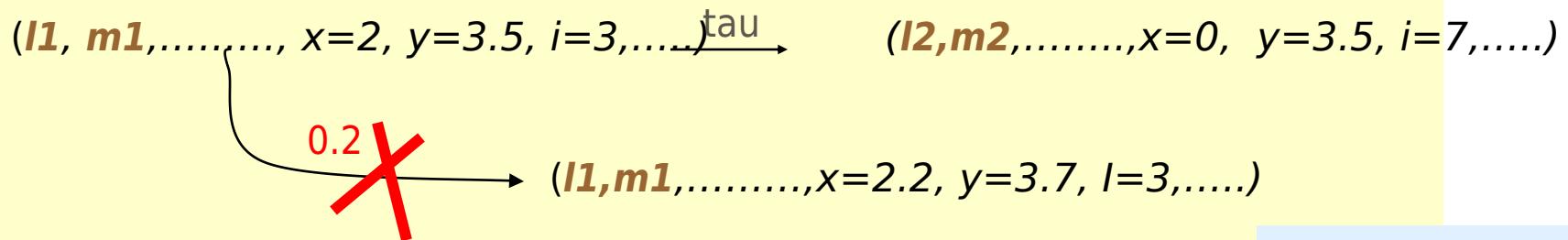
- ▶ Ordinary clocks ..... **x rate 1**
- ▶ Integer variables .... **x rate 0**
- ◀ Stopwatches ..... **x rate 0 or x rate 1** (*loc.dep.*)  
*Cassez, Larsen*
- ◀ Const. slope clocks .. **x rate n** where n is in Nat
- ◀ Parameters ..... **x rate 0** (and NOT assignable)
- ◀ Multirate clocks  
Lin. Hyb. Aut. ..... **x rate [l,u]** where l,u is in Nat  
*HyTech*  
linear guards & linear asgn.

# The UPPAAL Model

= Networks of Timed Automata + Integer Variables +....



Example transitions



If a URGENT CHANNEL



# UPPAAL & LEGO MINDSTORM

## *DEMO*

# LEGO Mindstorms/RCX

⌚ Sensors: temperature, light, rotation, pressure.

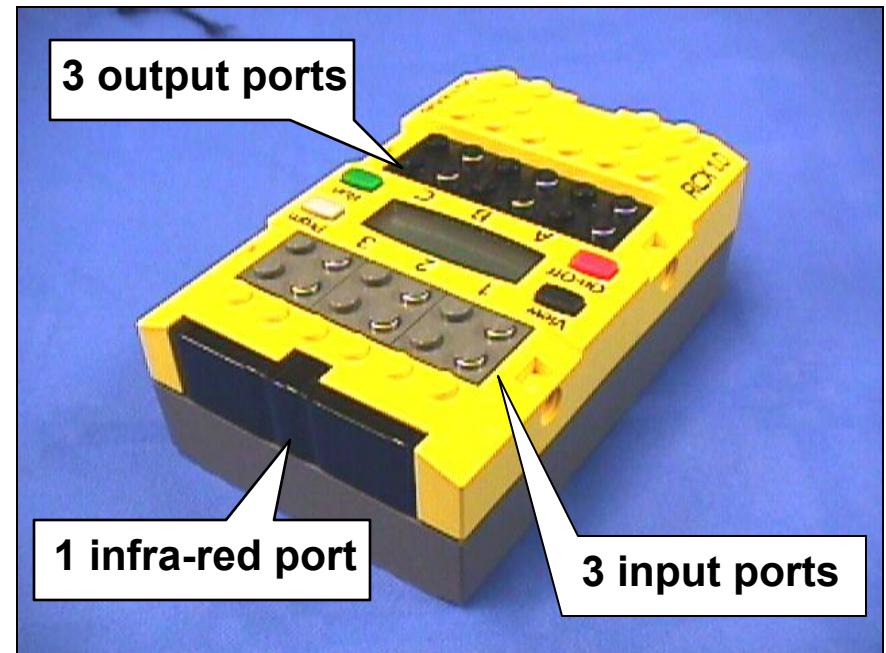
⌚ Actuators: motors, lamps,

⌚ Virtual machine:

⌚ 10 tasks, 4 timers,  
16 integers.

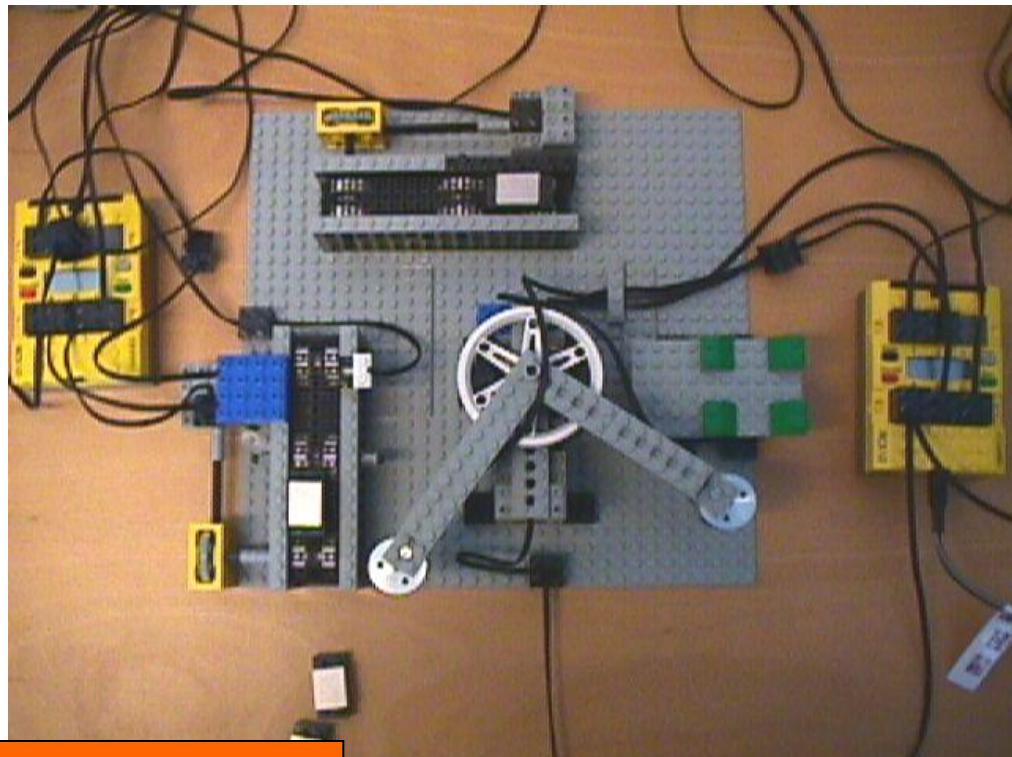
⌚ Several Programming Languages:

⌚ NotQuiteC, Mindstorm, Robotics, legOS, etc.

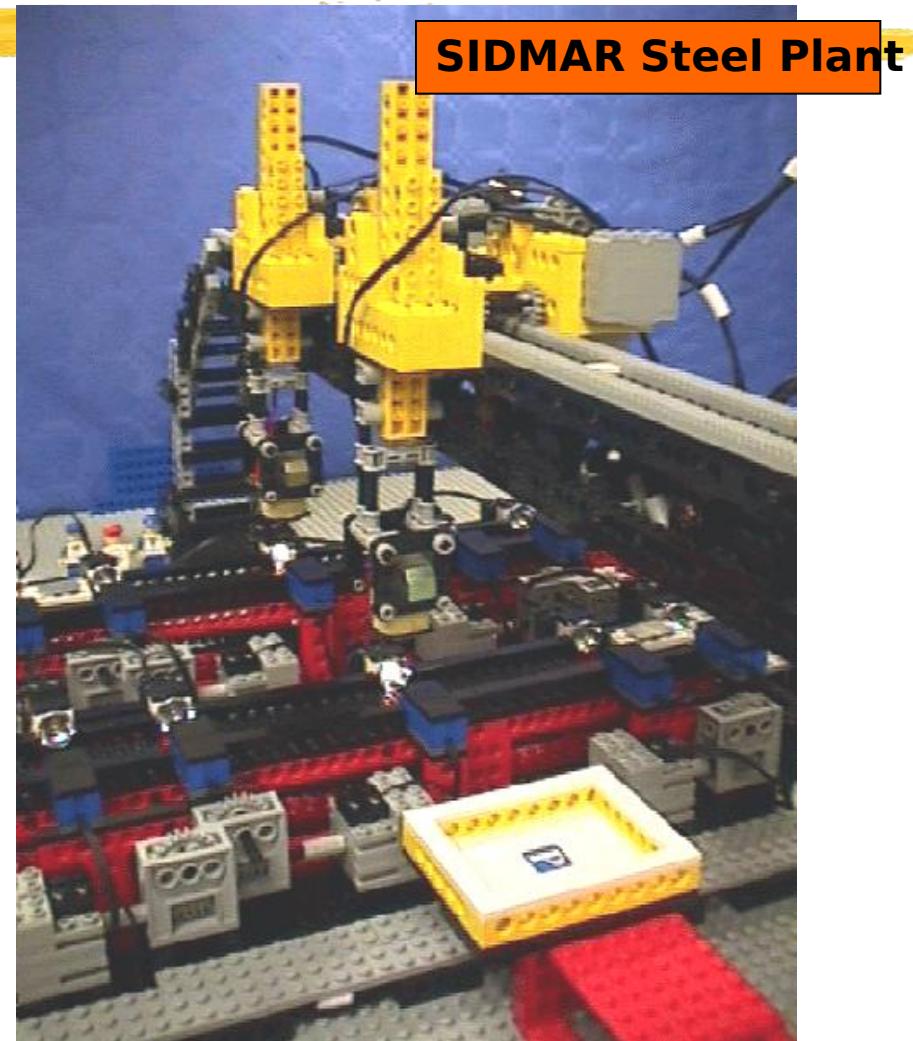


# LEGO Mindstorms/RCX

LEGO bricks!!!



Production Cell

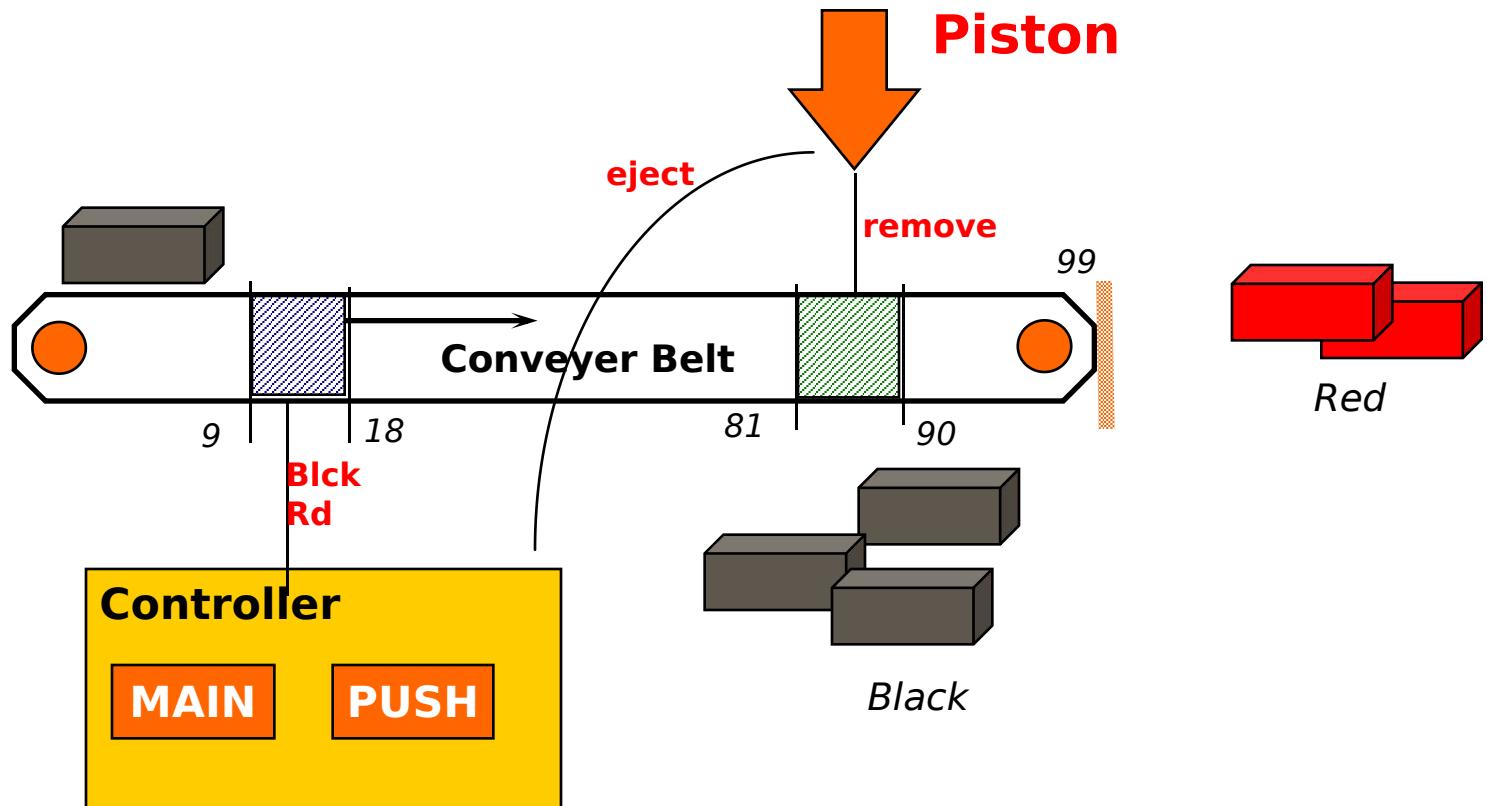
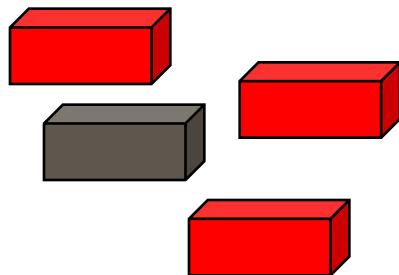


# First UPPAAL model

## *Sorting of Lego Boxes*

Ken Tindell

Boxes



**Exercise:** Design **Controller** so that only black boxes are being pushed out

# NQC programs

```
task MAIN{
    DELAY=75;
    LIGHT_LEVEL=35;
    active=0;
    Sensor(IN_1, IN_LIGHT);
    Fwd(OUT_A,1);
    Display(1);

    start PUSH;

    while(true){
        wait(IN_1<=LIGHT_LEVEL);
        ClearTimer(1);
        active=1;
        PlaySound(1);
        wait(IN_1>LIGHT_LEVEL);
    }
}
```

```
int active;
int DELAY;
int LIGHT_LEVEL;
```

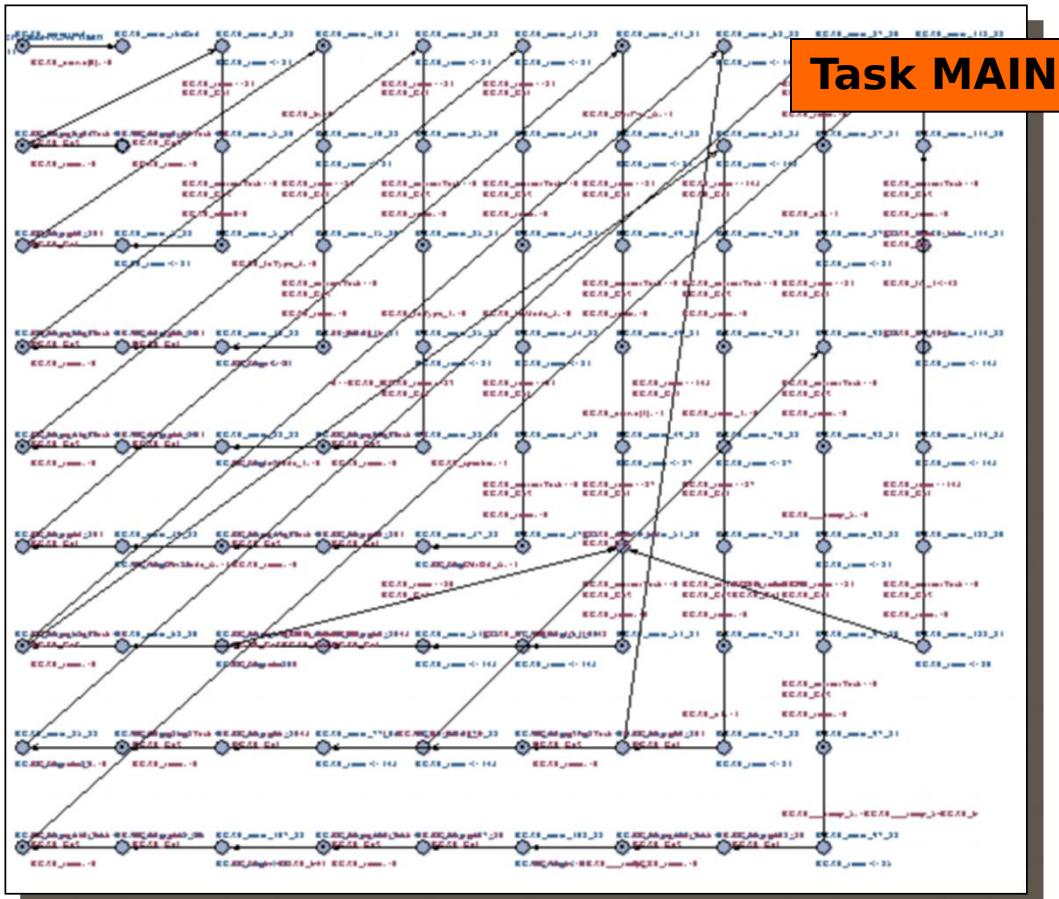
```
task PUSH{
    while(true) {
        wait(Timer(1)>DELAY && active==1);
        active=0;
        Rev(OUT_C,1);
        Sleep(8);
        Fwd(OUT_C,1);
        Sleep(12);
        Off(OUT_C);
    }
}
```



# **UPPAAL Demo**

# From RCX to UPPAAL

- Model includes Round-Robin Scheduler.
  - Compilation of RCX tasks into TA models.
  - Presented at ECRTS 2000 (yesterday) in Stockholm.





# THE UPPAAL ENGINE

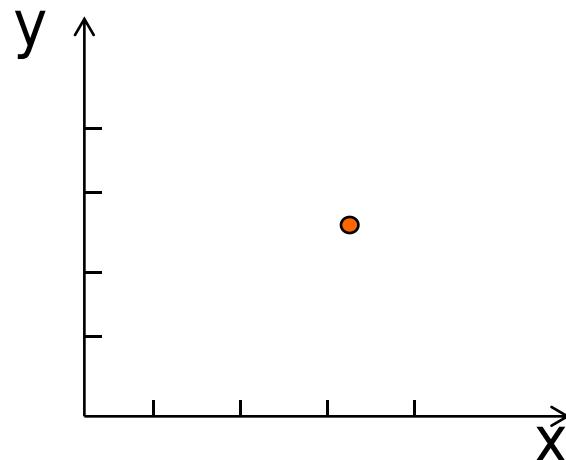
*Symbolic  
Reachability  
Checking*

# Zones

## From infinite to finite

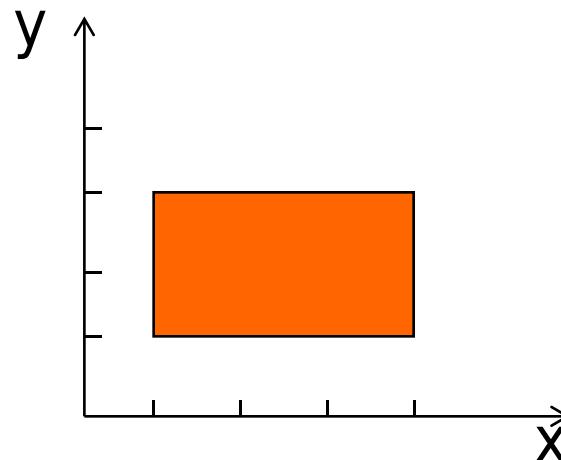
State

(n,  $x=3.2, y=2.5$ )



Symbolic state (set)

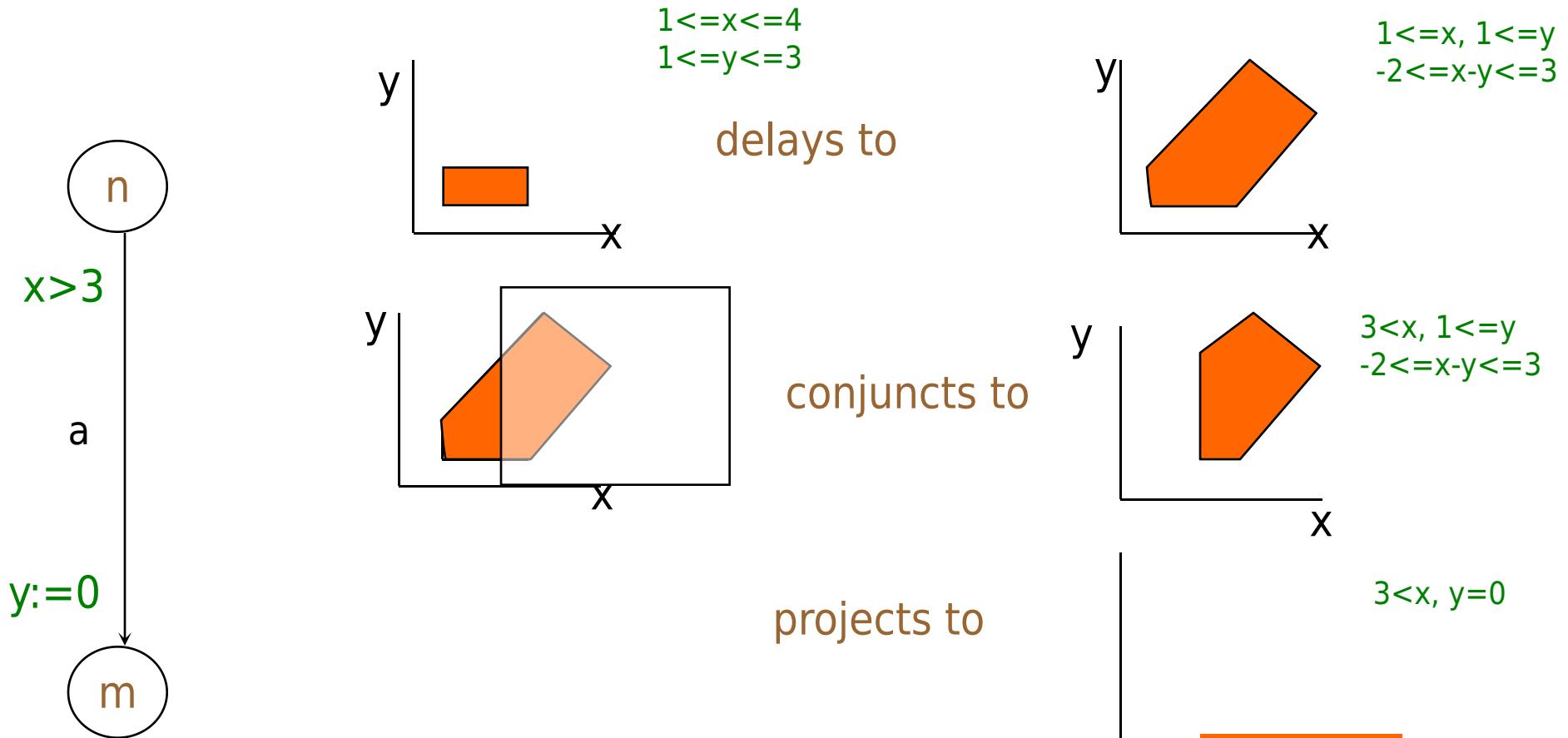
(n,  $1 \leq x \leq 4, 1 \leq y \leq 3$ )



Zone:

conjunction of  
 $x-y \leq n, x \geq n$

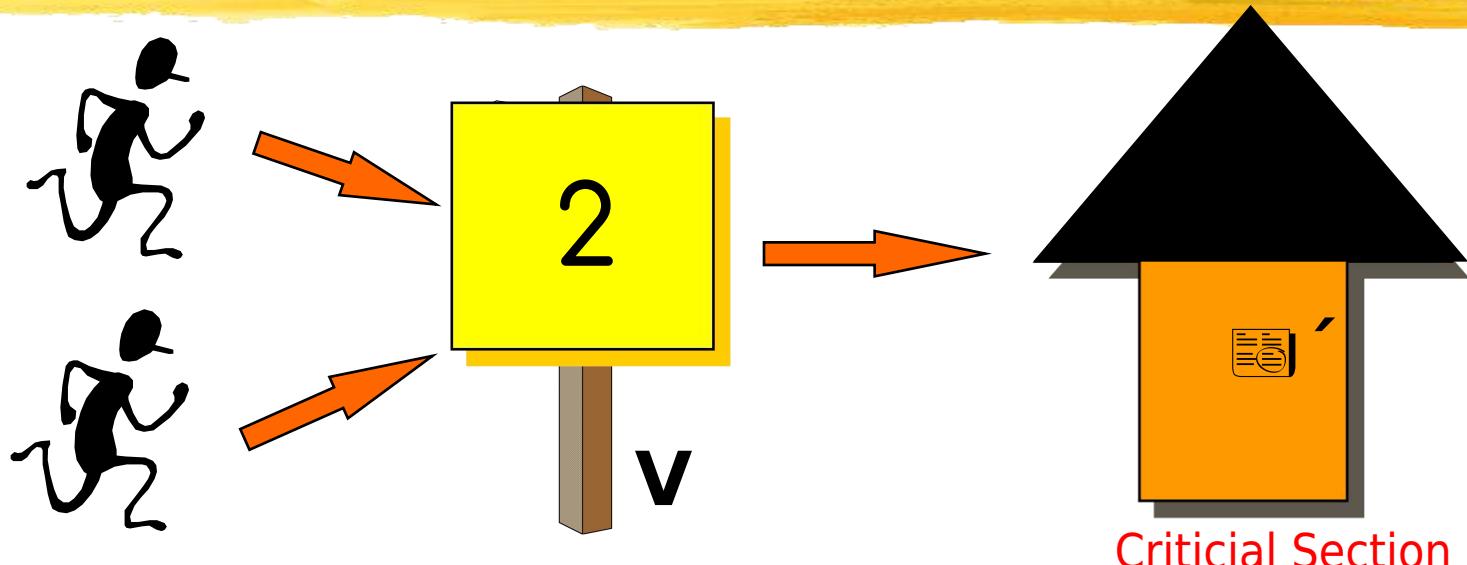
# Symbolic Transitions



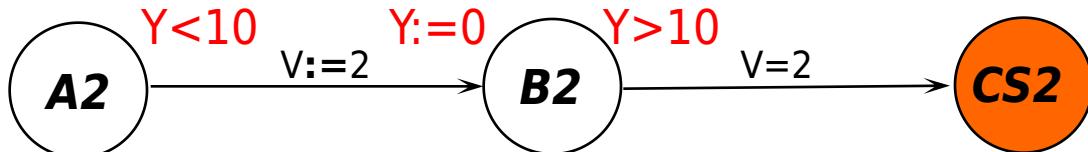
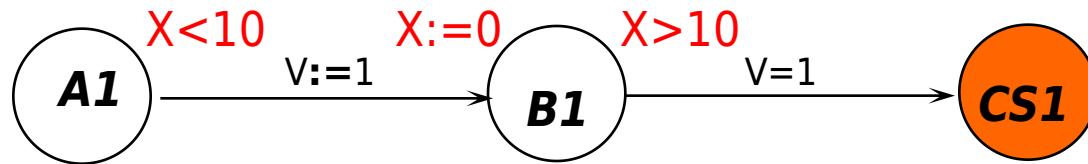
Thus  $(n, 1 \leq x \leq 4, 1 \leq y \leq 3) = a \Rightarrow (m, 3 < x, y = 0)$

# Fischer's Protocol

## *analysis using zones*

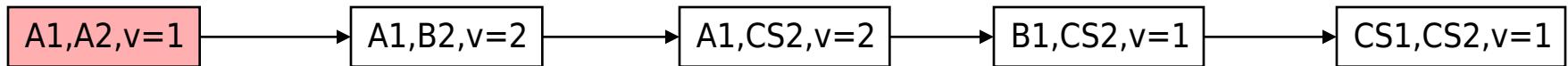
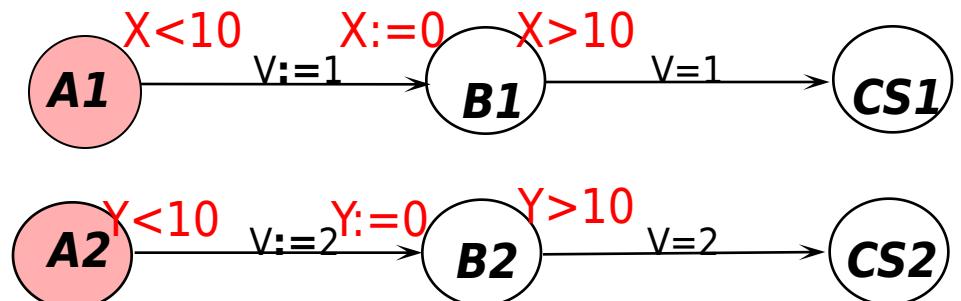


**Init**  
 $V=1$



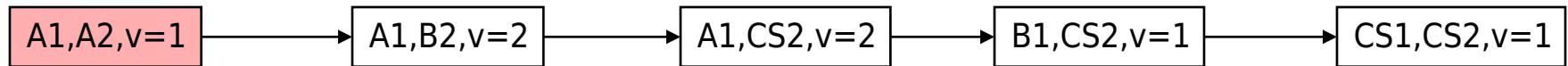
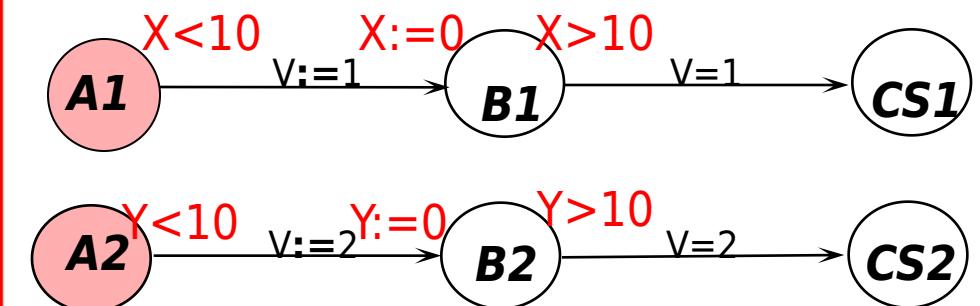
# Fischers cont.

*Untimed case*

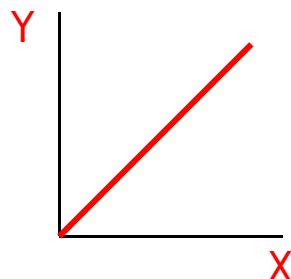


# Fischers cont.

*Untimed case*

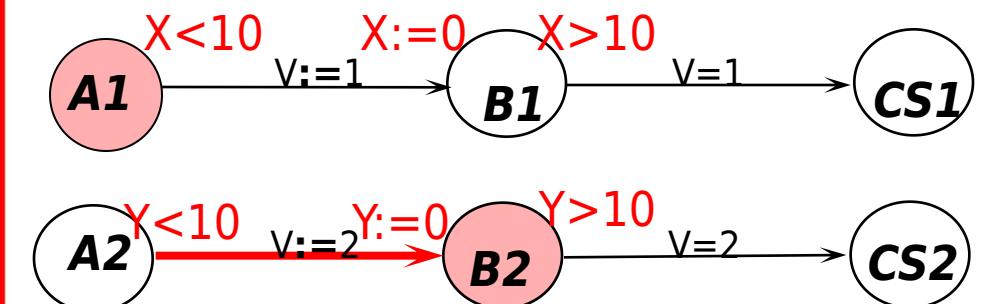


*Taking time into account*

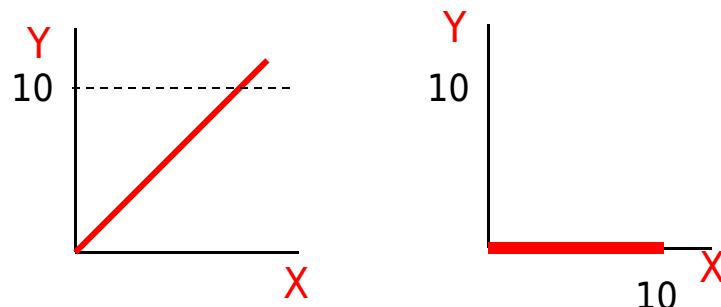


# Fischers cont.

*Untimed case*

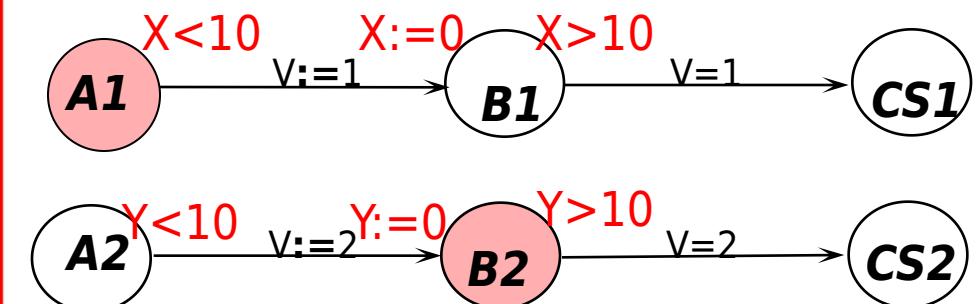


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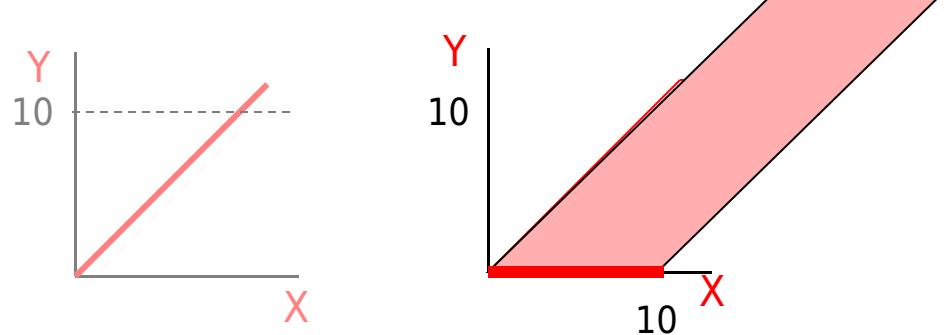


# Fischers cont.

*Untimed case*

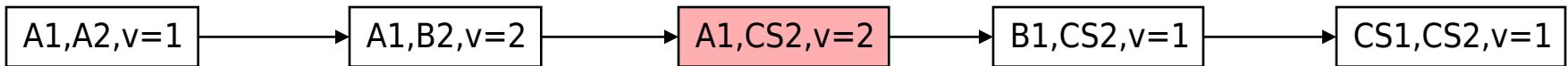
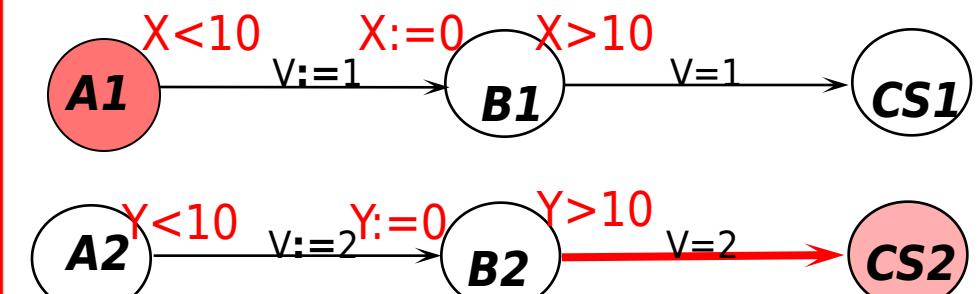


*Taking time into account*

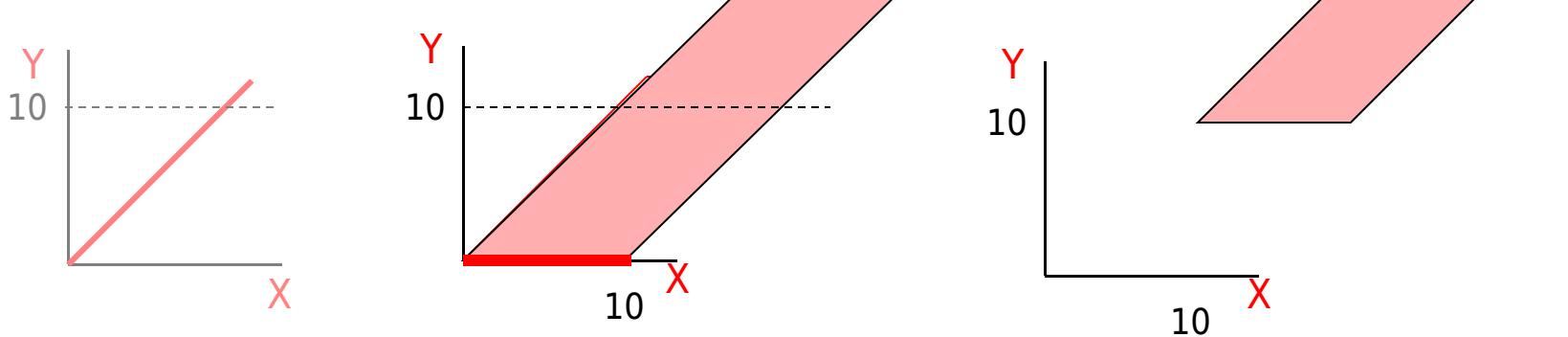


# Fischers cont.

*Untimed case*

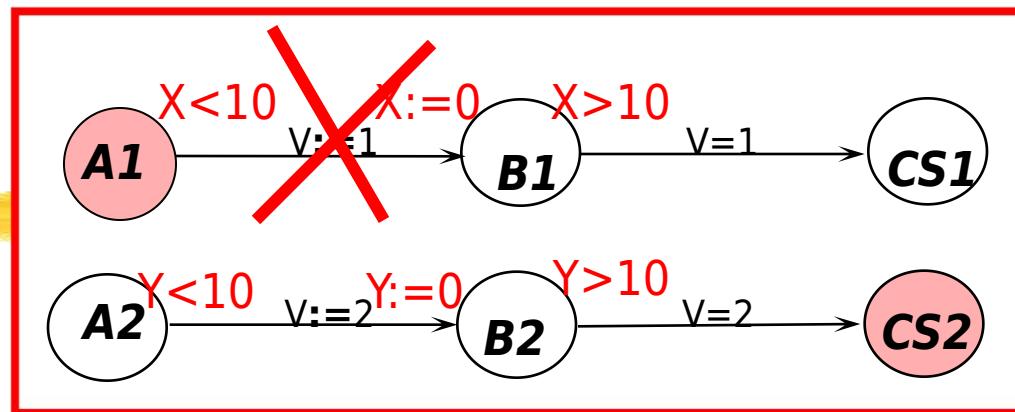


*Taking time into account*

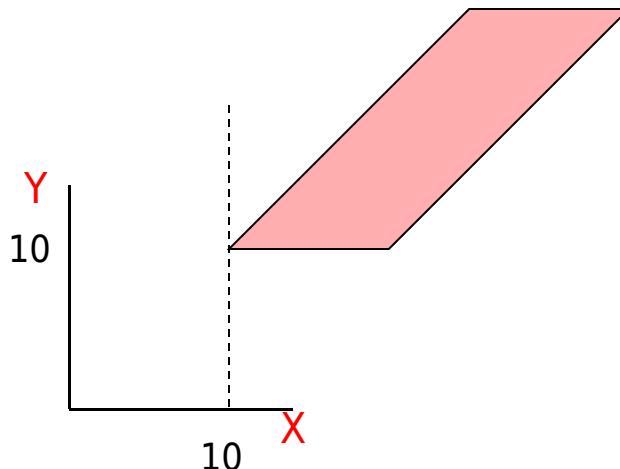
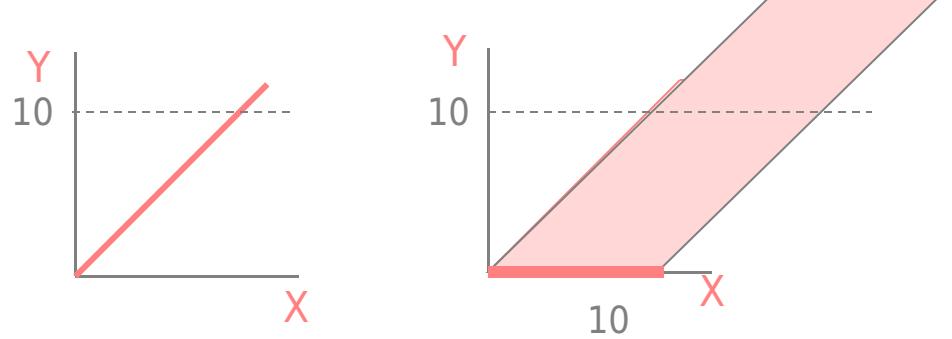


# Fischers cont.

*Untimed case*



*Taking time into account*



# Canonical Datastructures for Zones

## *Difference Bounded Matrices*

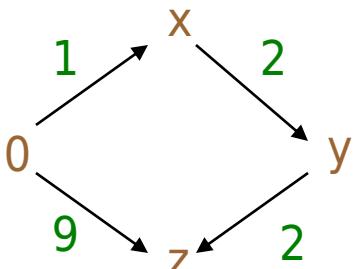
Bellman 1958, Dill 1989

### Inclusion

**D1**

$x \leq 1$   
 $y - x \leq 2$   
 $z - y \leq 2$   
 $z \leq 9$

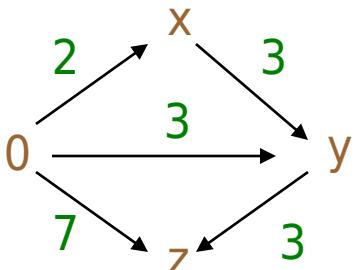
**Graph**



**D2**

$x \leq 2$   
 $y - x \leq 3$   
 $y \leq 3$   
 $z - y \leq 3$   
 $z \leq 7$

**Graph**



$$? \subseteq ?$$

# Canonical Datastructures for Zones

## *Difference Bounded Matrices*

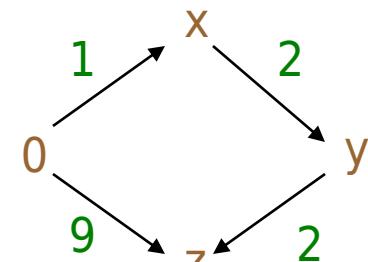
Bellman 1958, Dill 1989

### Inclusion

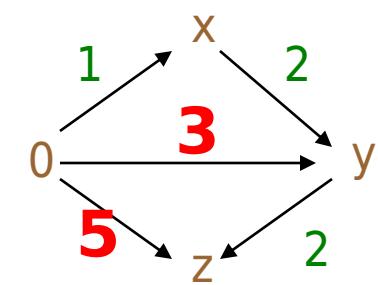
**D1**

$x \leq 1$   
 $y - x \leq 2$   
 $z - y \leq 2$   
 $z \leq 9$

**Graph**



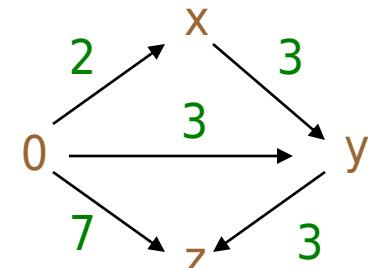
Shortest  
Path  
Closure



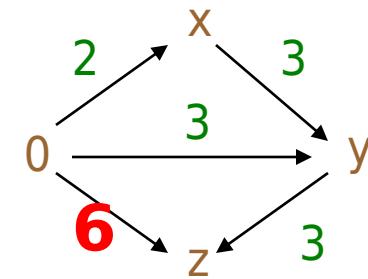
**D2**

$x \leq 2$   
 $y - x \leq 3$   
 $y \leq 3$   
 $z - y \leq 3$   
 $z \leq 7$

**Graph**



Shortest  
Path  
Closure



# Canonical Datastructures for Zones

## *Difference Bounded Matrices*

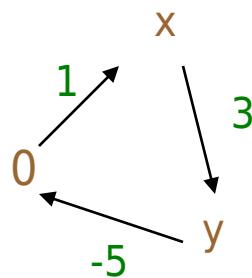
Bellman 1958, Dill 1989

### Emptiness

**D**

$$\begin{array}{l} x \leq 1 \\ y >= 5 \\ y - x \leq 3 \end{array}$$

**Graph**



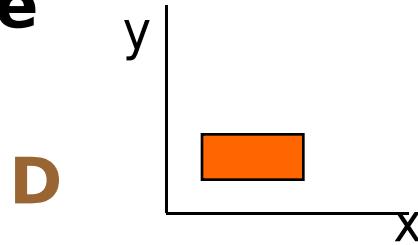
Compact

**Negative Cycle  
iff  
empty solution set**

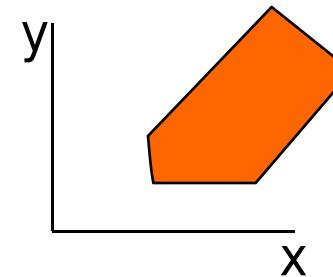
# Canonical Datastructures for Zones

## *Difference Bounded Matrices*

**Future**

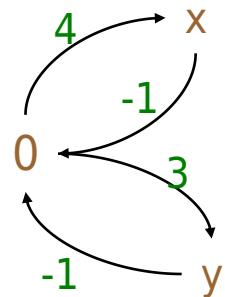


$$\begin{aligned} 1 \leq x \leq 4 \\ 1 \leq y \leq 3 \end{aligned}$$

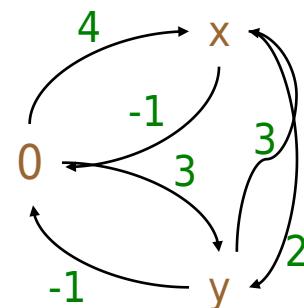


**Future D**

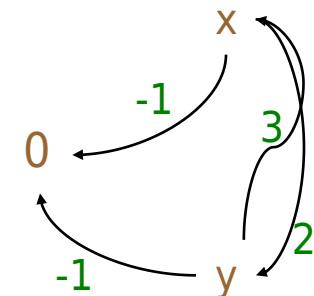
$$\begin{aligned} 1 \leq x, 1 \leq y \\ -2 \leq x-y \leq 3 \end{aligned}$$



Shortest  
Path Closure



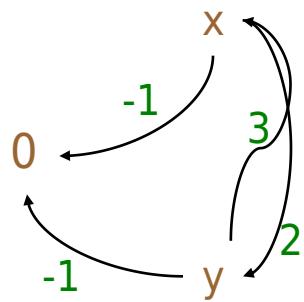
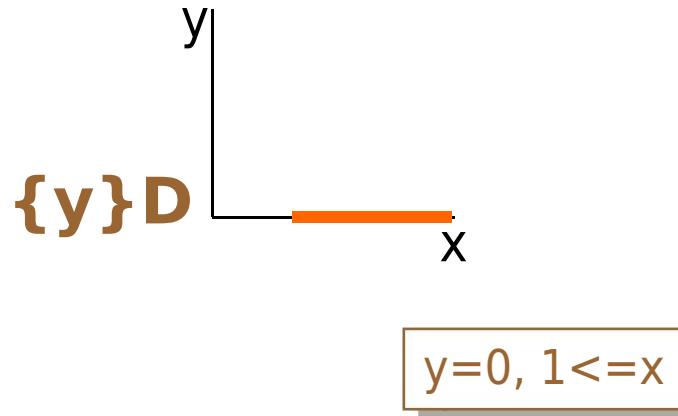
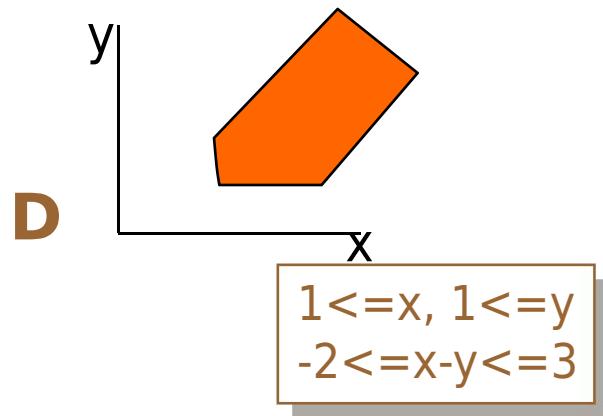
Remove  
upper  
bounds  
on clocks



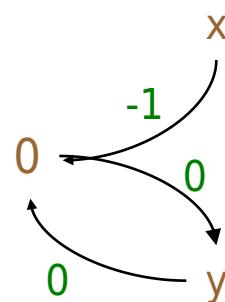
# Canonical Datastructures for Zones

## *Difference Bounded Matrices*

### Reset



Remove all  
bounds  
involving y  
and set y to 0

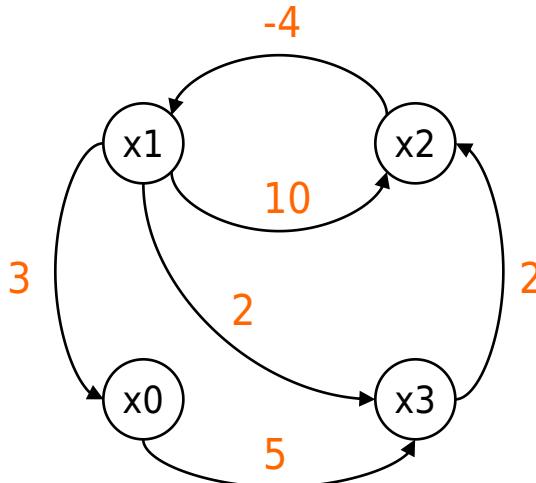


# Improved Datastructures

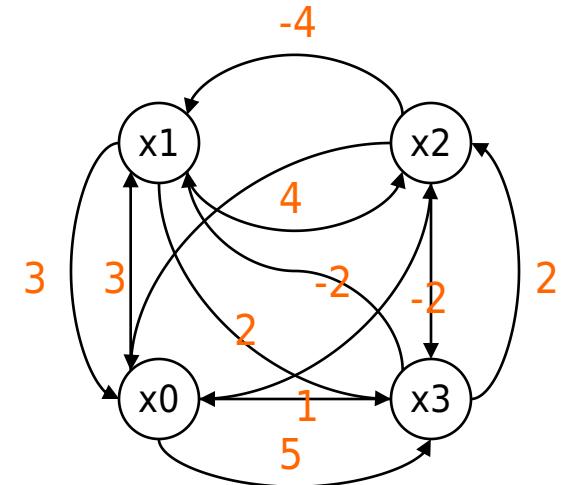
## Compact Datastructure for Zones

RTSS 1997

```
x1-x2<=4
x2-x1<=10
x3-x1<=2
x2-x3<=2
x0-x1<=3
x3-x0<=5
```



**Shortest  
Path  
Closure  
 $O(n^3)$**

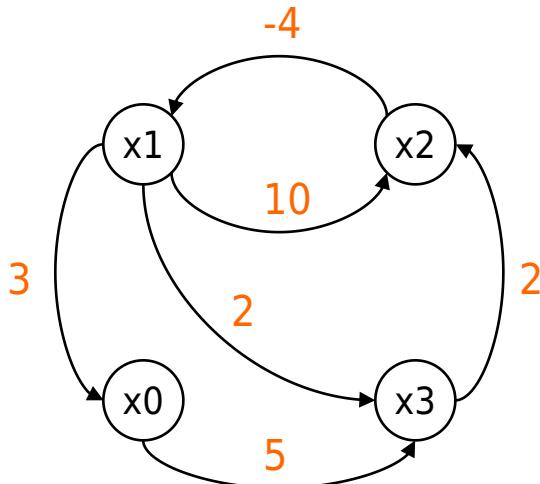


# Improved Datastructures

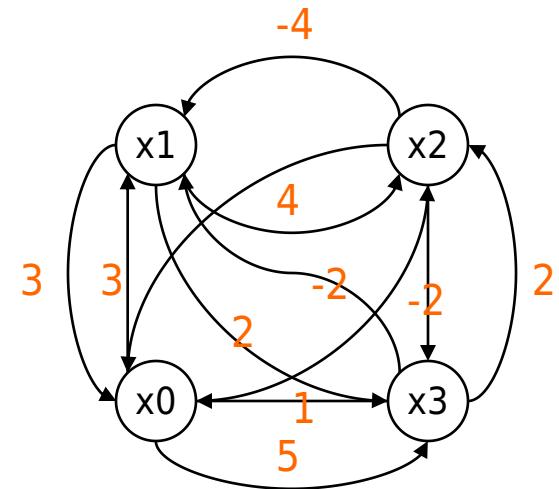
## Compact Datastructure for Zones

RTSS 1997

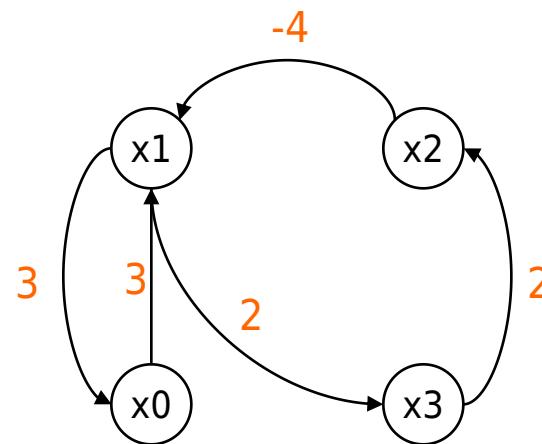
```
x1-x2<=4
x2-x1<=10
x3-x1<=2
x2-x3<=2
x0-x1<=3
x3-x0<=5
```



**Shortest Path Closure**  
 $O(n^3)$



**Shortest Path Reduction**  
 $O(n^3)$

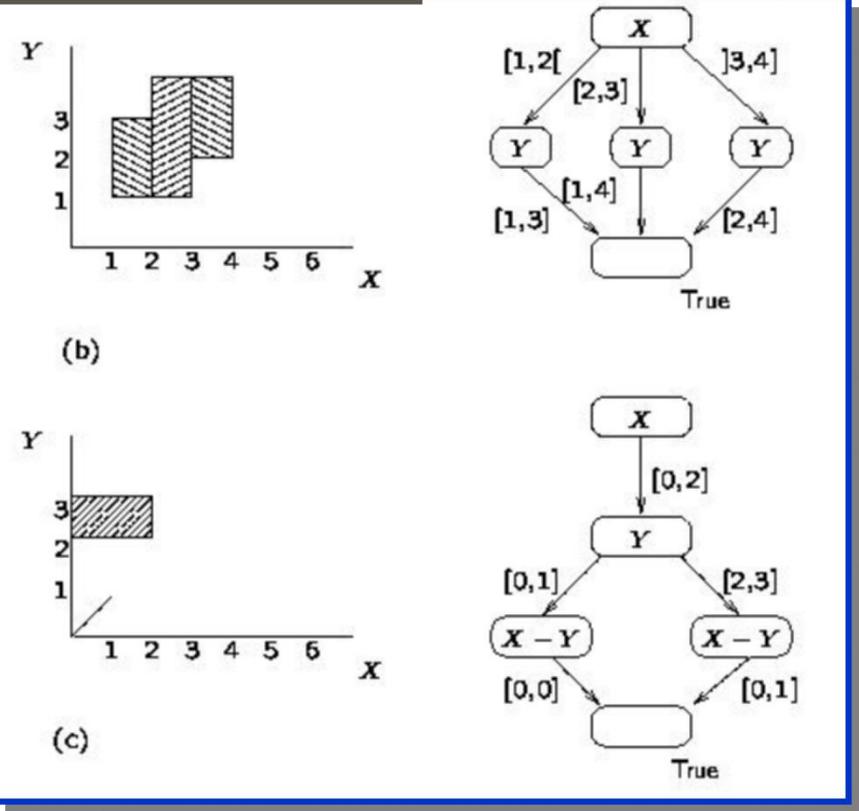


Canonical wrt =  
Space worst  $O(n^2)$   
practice  $O(n)$

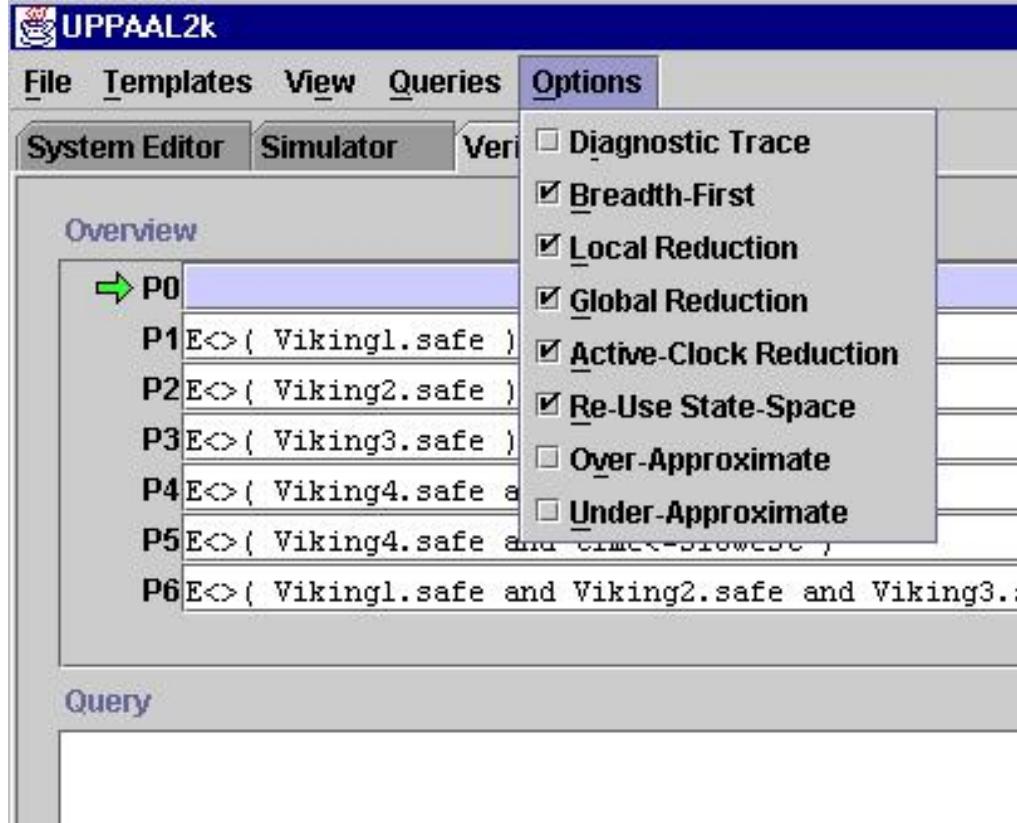
# Other Symbolic Datastructures

- ⌚ Regions Alur, Dill
- ⌚ NDD's Maler et. al.
- ⌚ CDD's UPPAAL/CAV99
- ⌚ DDD's Møller, Lichtenberg
- ⌚ Polyhedra HyTech
- ⌚ .....

## CDD-representations



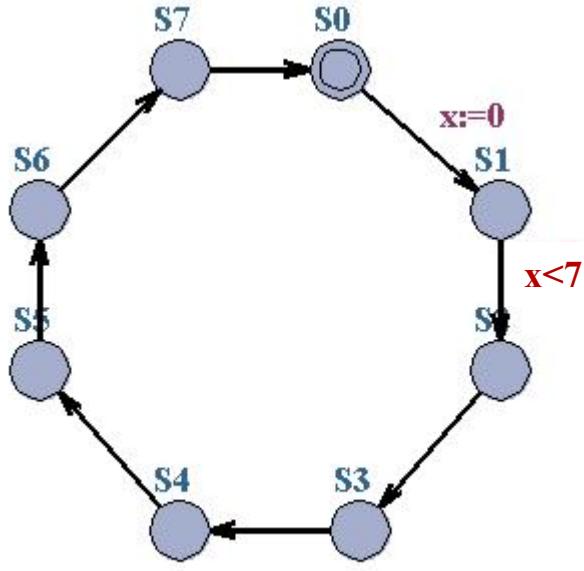
# Verification Options



## Case Studies

# Representation of symbolic states

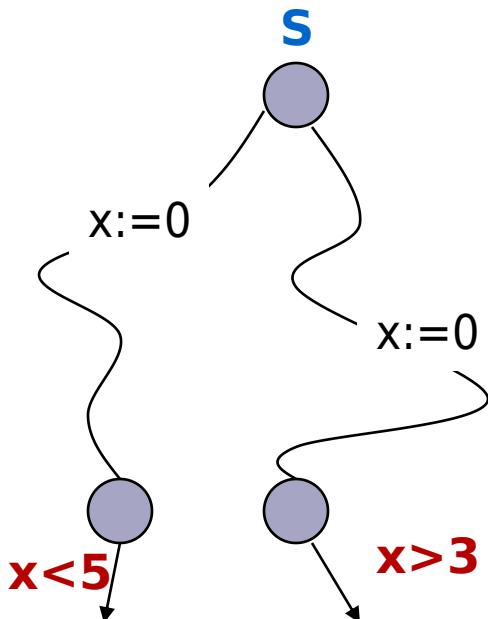
## *(In)Active Clock Reduction*



x is only **active** in location **S1**

### Definition

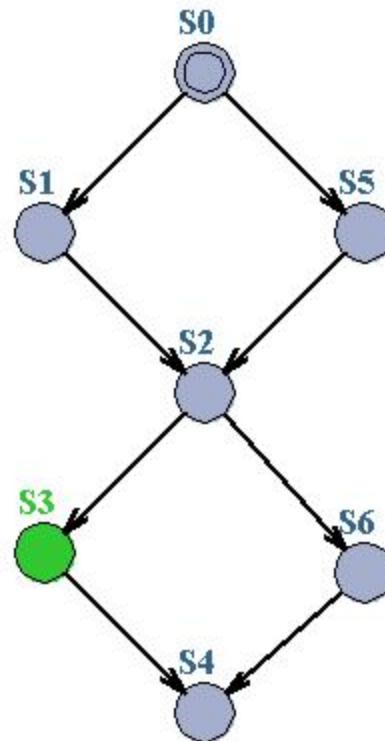
x is **inactive** at **S** if on all path from **S**, x is always reset before being tested.



### Case Studies

# When to store symbolic state

## *Global Reduction*



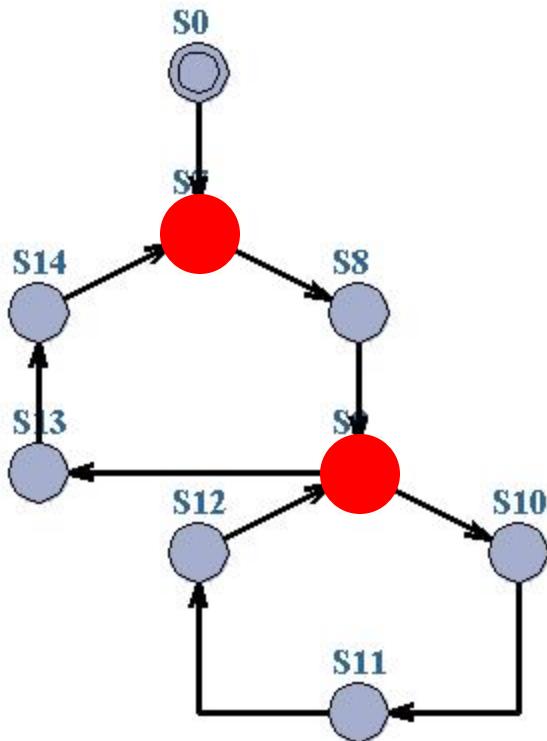
However,  
**Passed** list useful for  
efficiency

### Case Studies

**No Cycles:** **Passed** list not needed for *termination*

# When to store symbolic state

## *Global Reduction*

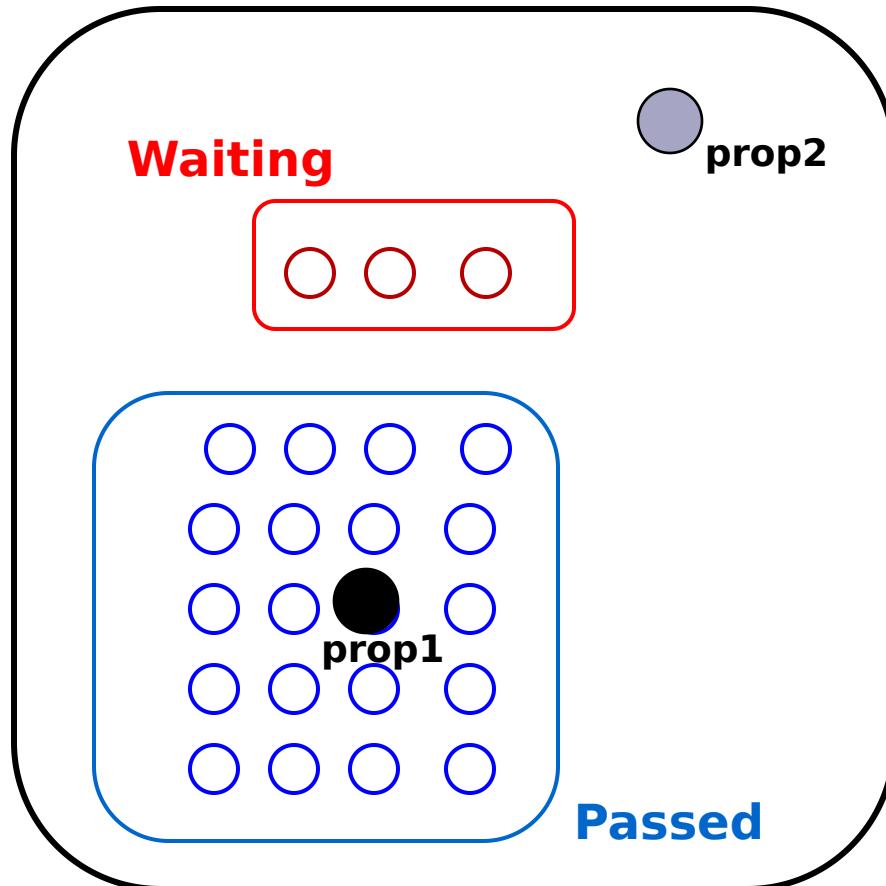


### Cycles:

Only symbolic states involving loop-entry points need to be saved on **Passed** list

### Case Studies

# Reuse State Space



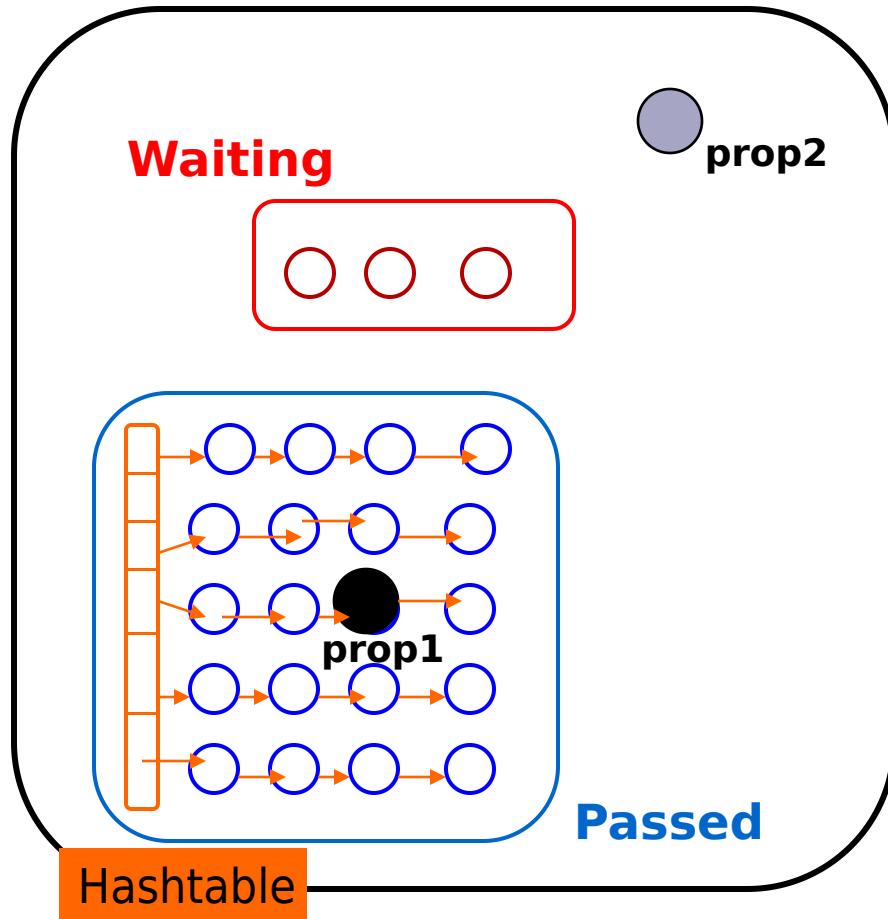
$A[]$  prop1  
 $A[]$  prop2  
 $A[]$  prop3  
 $A[]$  prop4  
 $A[]$  prop5  
 .  
 .  
 .  
 $A[]$  propn

Search in existing **Passed** list before continuing search

Which order to search?

## Case Studies

# Reuse State Space



$A[]$  prop1  
 $A[]$  prop2  
 $A[]$  prop3  
 $A[]$  prop4  
 $A[]$  prop5  
 .  
 .  
 .  
 $A[]$  propn

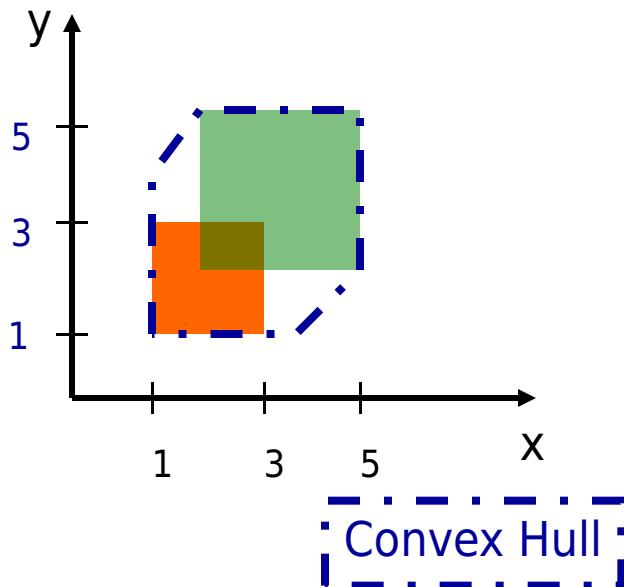
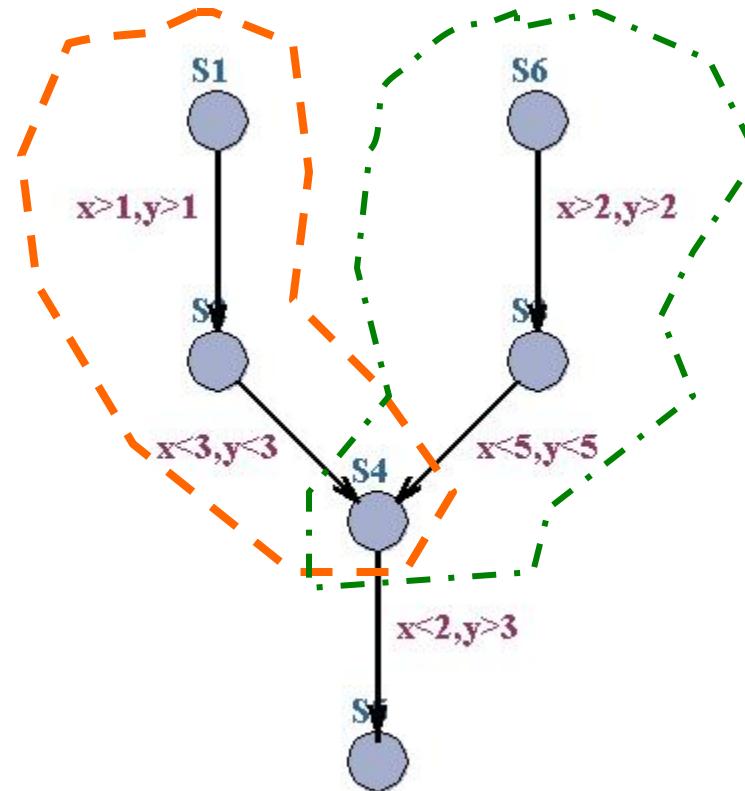
Search in existing **Passed** list before continuing search

Which order to search?

## Case Studies

# Over-approximation

## *Convex Hull*



### Case Studies



# **Case Studies**

# Case Studies: Protocols

- ⌚ Philips Audio Protocol [HS'95, CAV'95, RTSS'95, CAV'96]
- ⌚ Collision-Avoidance Protocol [SPIN'95]
- ⌚ Bounded Retransmission Protocol [TACAS'97]
- ⌚ Bang & Olufsen Audio/Video Protocol [RTSS'97]
- ⌚ TDMA Protocol [PRFTS'97]
- ⌚ Lip-Synchronization Protocol [FMICS'97]
- ⌚ Multimedia Streams [DSVIS'98]
- ⌚ ATM ABR Protocol [CAV'99]
- ⌚ ABB Fieldbus Protocol [ECRTS'2k]
- ⌚ IEEE 1394 Firewire Root Contention (2000)

# Case-Studies: Controllers



- ◉ Gearbox Controller [TACAS'98]
- ◉ Bang & Olufsen Power Controller  
[RTPS'99, FTRTFT'2k]
- ◉ SIDMAR Steel Production Plant [RTCSA'99, DSVV'2k]
- ◉ Real-Time RCX Control-Programs [ECRTS'2k]
- ◉ Experimental Batch Plant (2000)
- ◉ RCX Production Cell (2000)

# Steel Production Plant

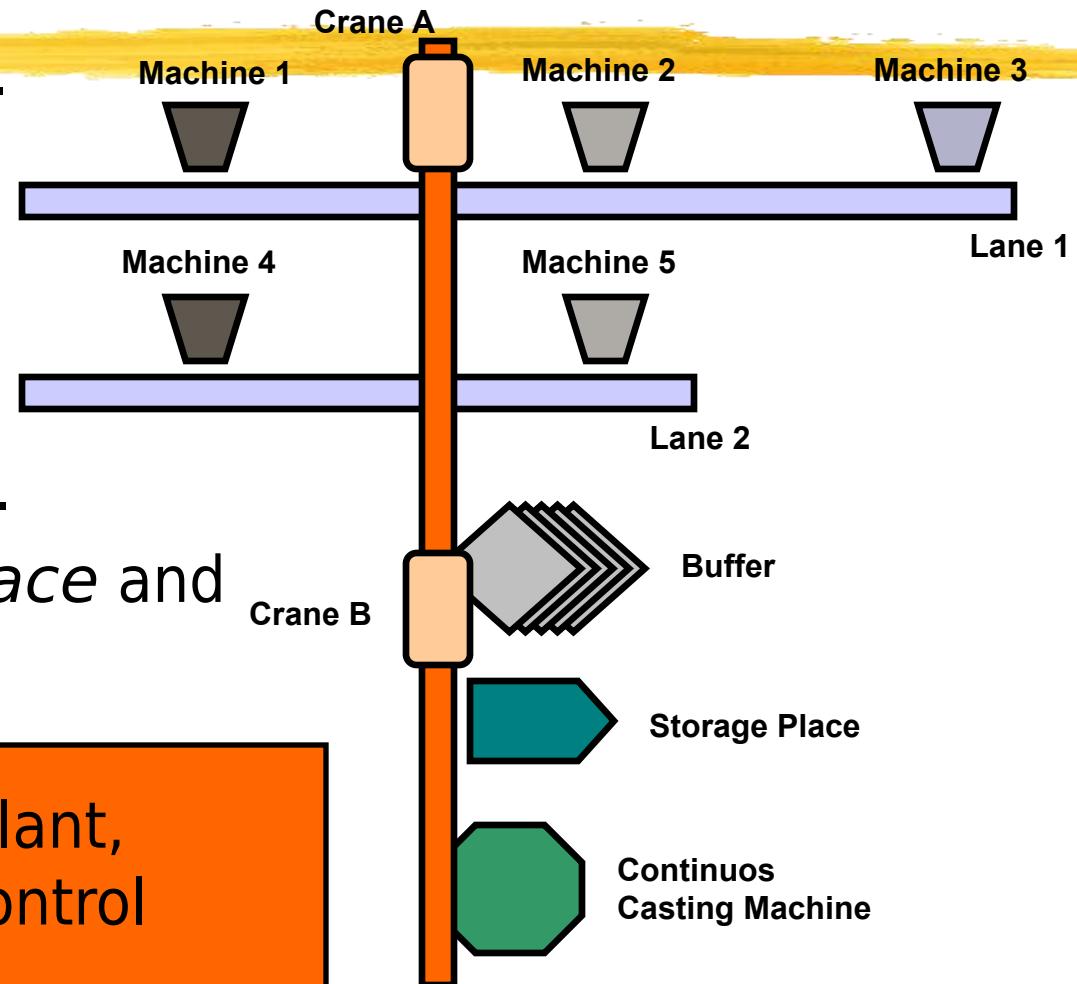
✉ A. Fehnker, T. Hune, K. G. Larsen, P. Pettersson

✉ Case study of Esprit-LTR project 26270 VHS

✉ Physical plant of SIDMAR located in Gent, Belgium.

✉ Part between *blast furnace* and *hot rolling mill*.

**Objective:** **model** the plant,  
obtain **schedule** and control  
**program** for plant.



# Steel Production Plant

**Input:** sequence of steel loads ("pigs").



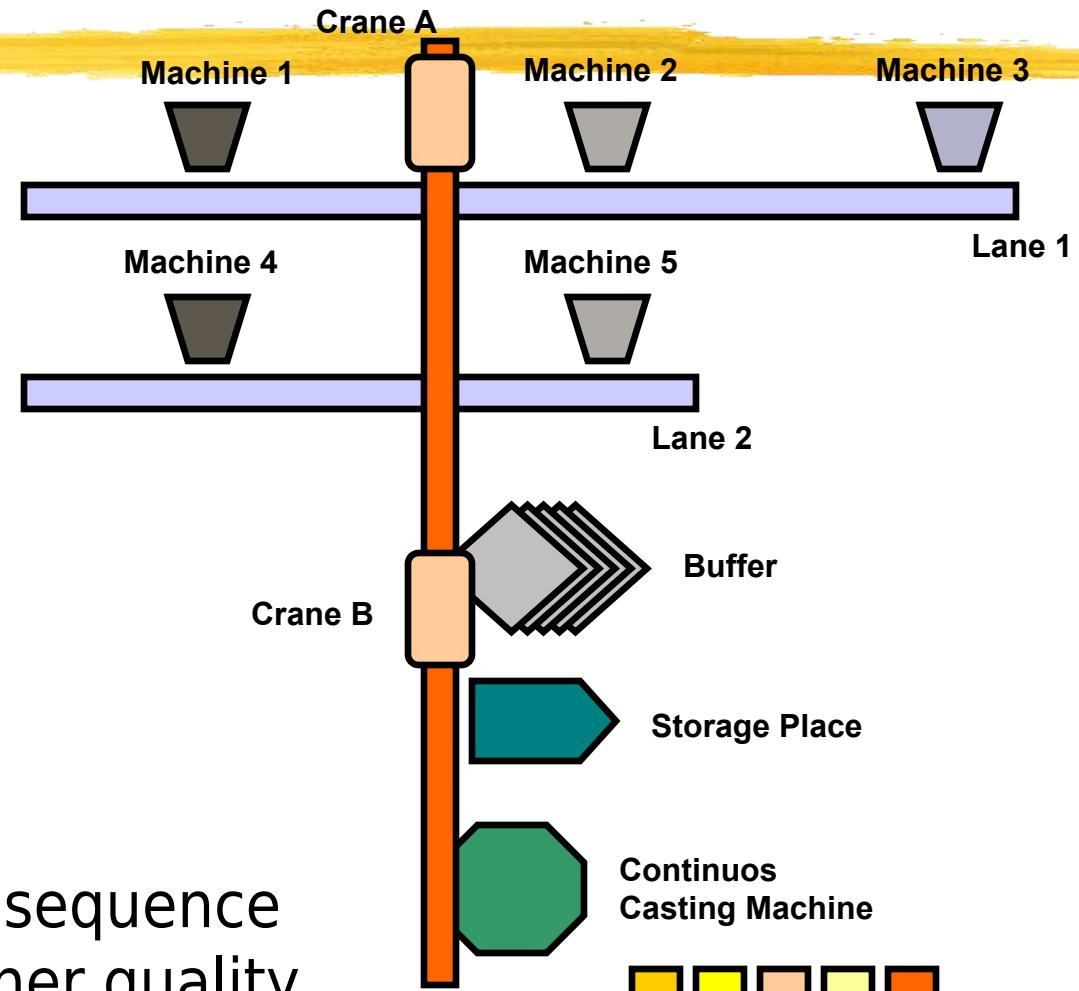
Load follows **Recipe** to become certain quality,  
e.g:

start; **T1@10; T2@20;**

**T3@10; T2@10;**

end within 120.

**Output:** sequence of higher quality steel.



# Steel Production Plant

**Input:** sequence of steel loads ("pigs").



Load follows **Recipe** to become certain quality,  
e.g:

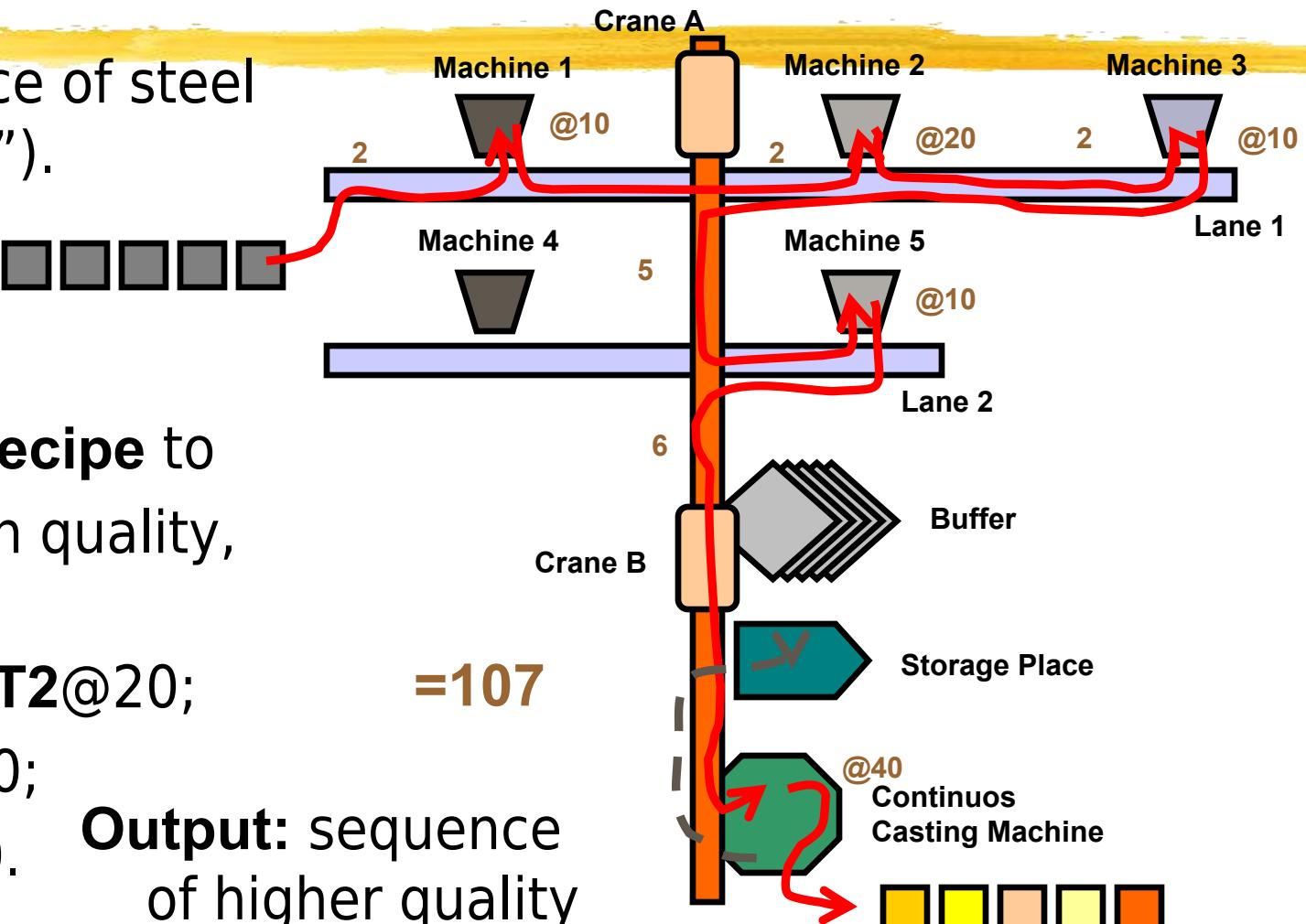
start; **T1@10; T2@20;**

=107

**T3@10; T2@10;**

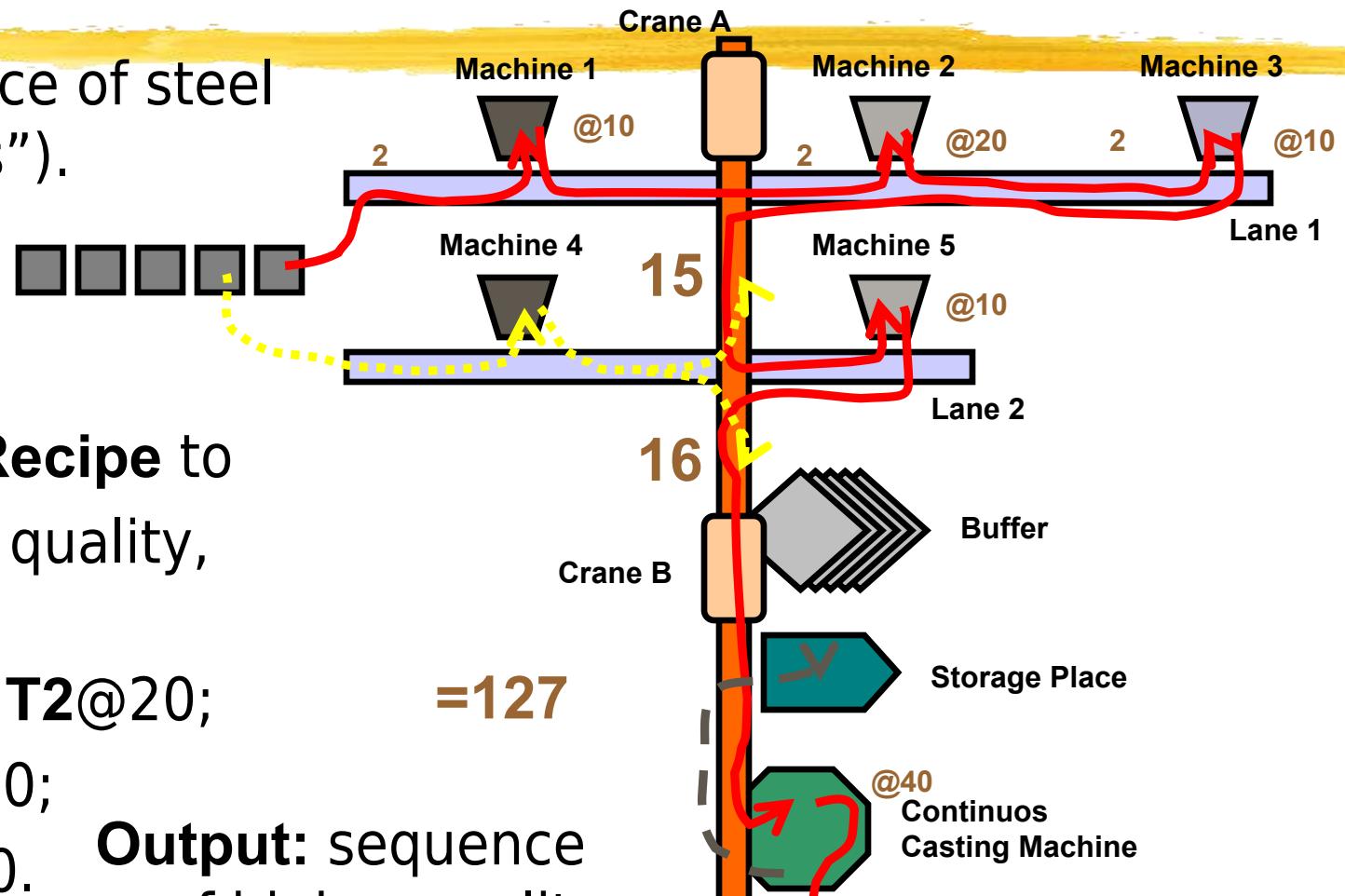
end within 120.

**Output:** sequence of higher quality steel.



# Steel Production Plant

**Input:** sequence of steel loads ("pigs").



Load follows **Recipe** to obtain certain quality,  
e.g:

start; **T1@10; T2@20;**

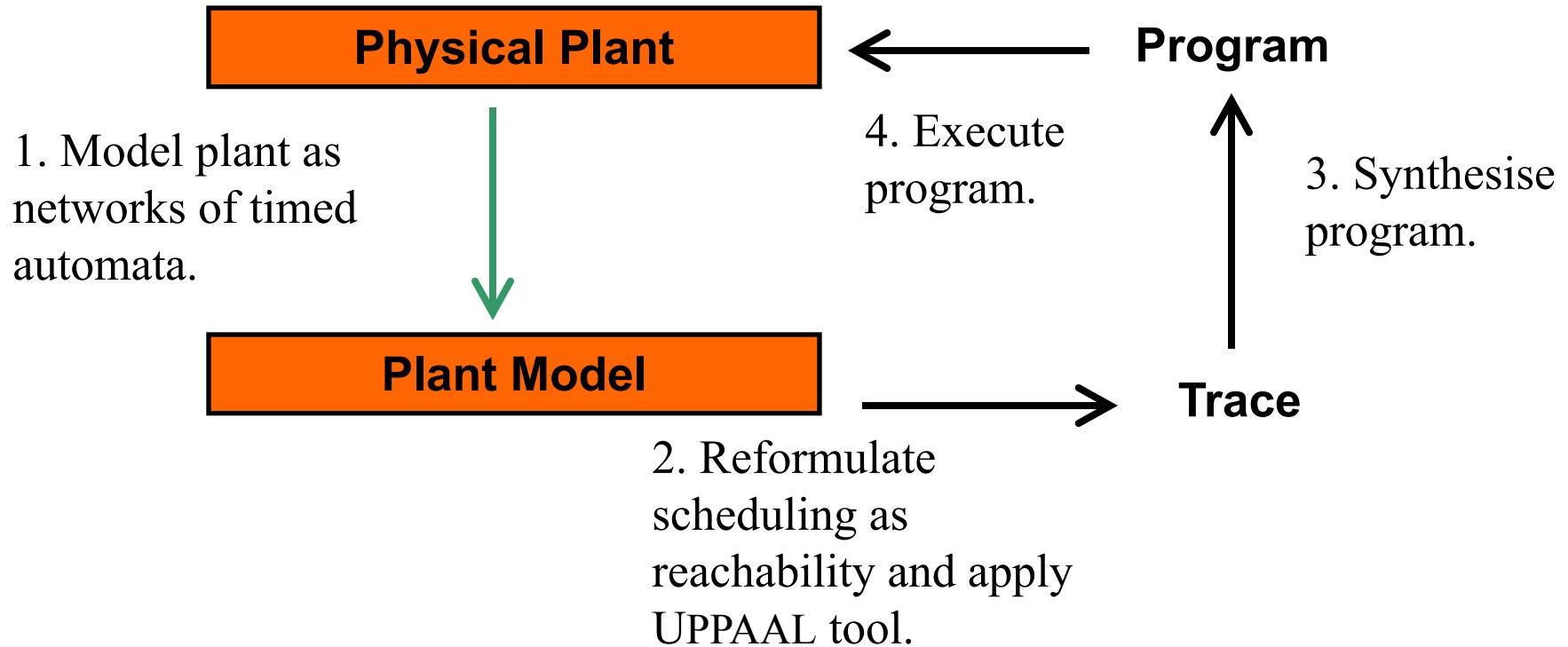
=127

**T3@10; T2@10;**

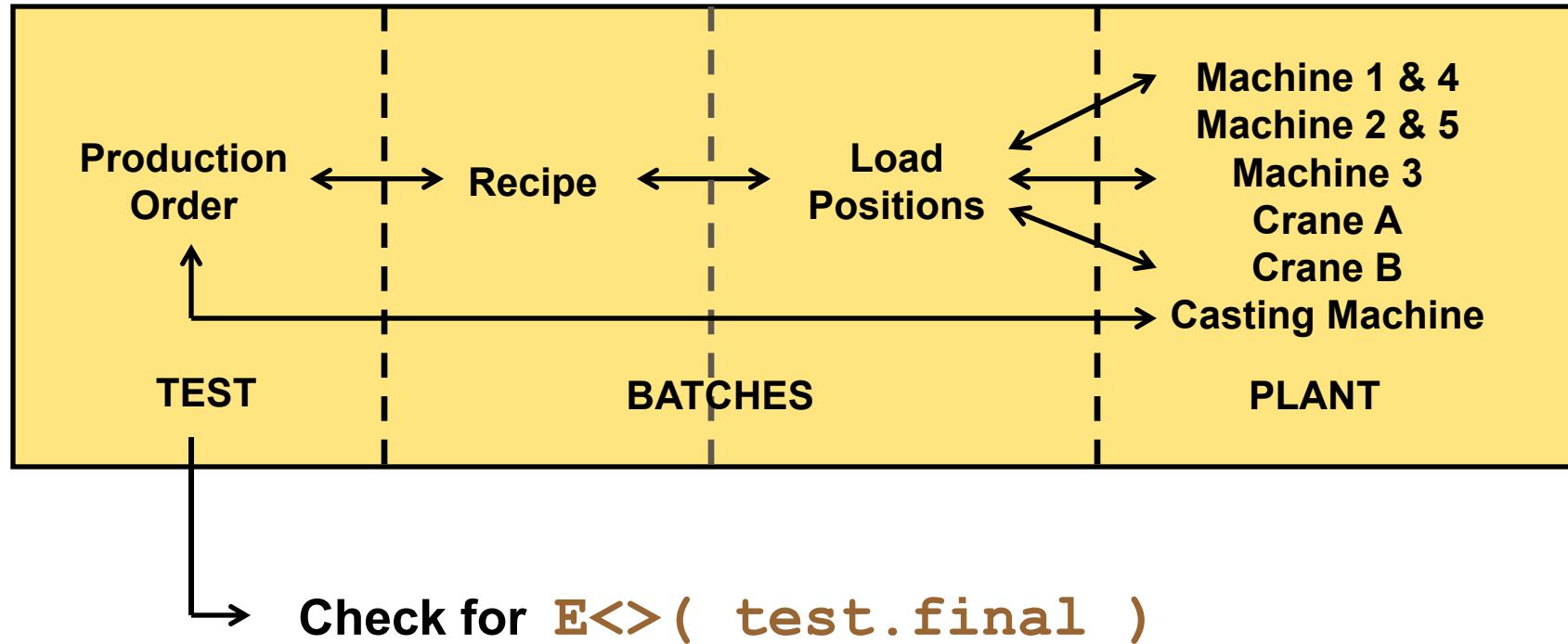
end within 120.

**Output:** sequence of higher quality steel.

# Modus Operandi



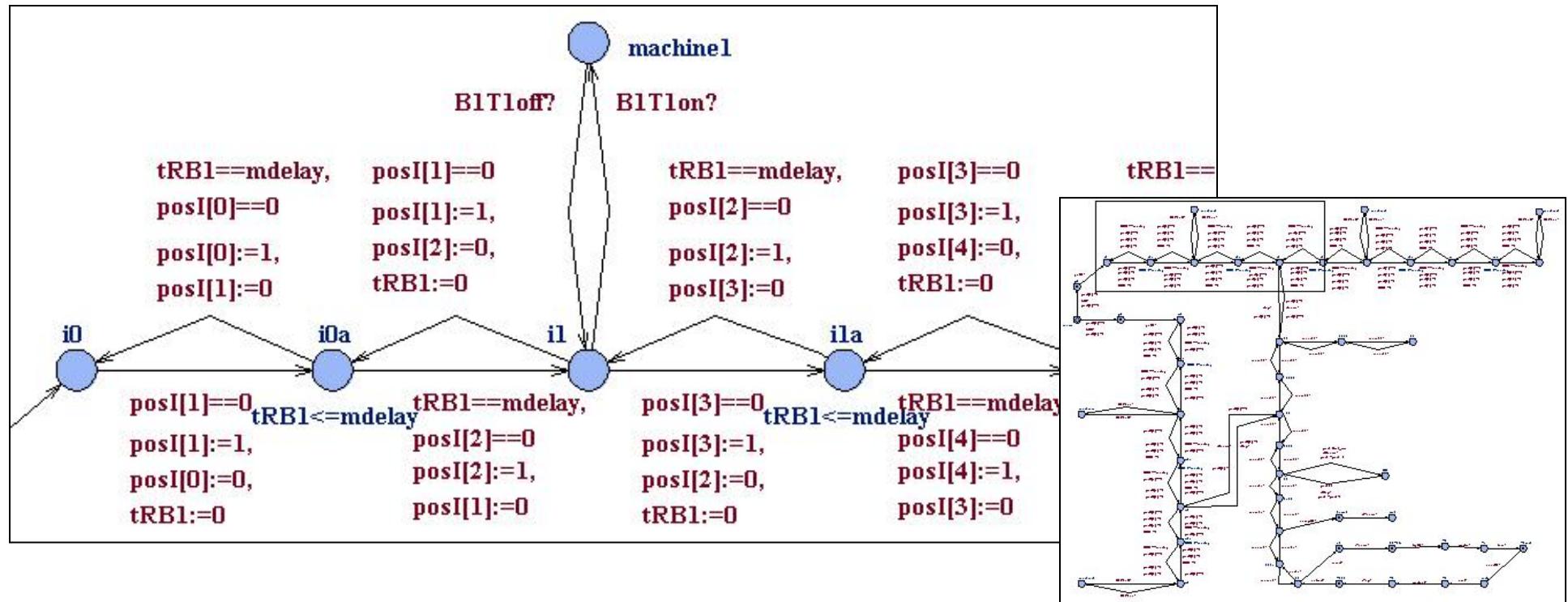
# Overview of Plant Model



UPPAAL generates diagnostic trace if property holds.

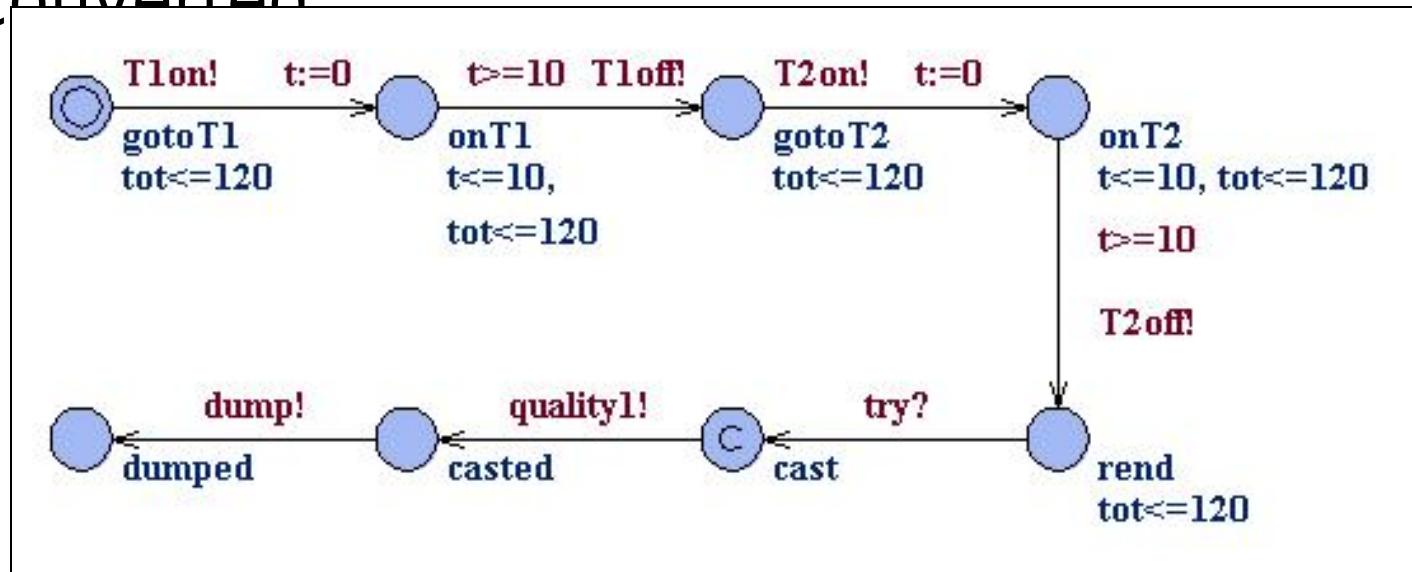
# Load Automaton

- ⌚ Models all possible movements of load.
- ⌚ Clock **tRB1** used to model time consumption.
- ⌚ Bit vectors **posI** and **posII** models mutex.



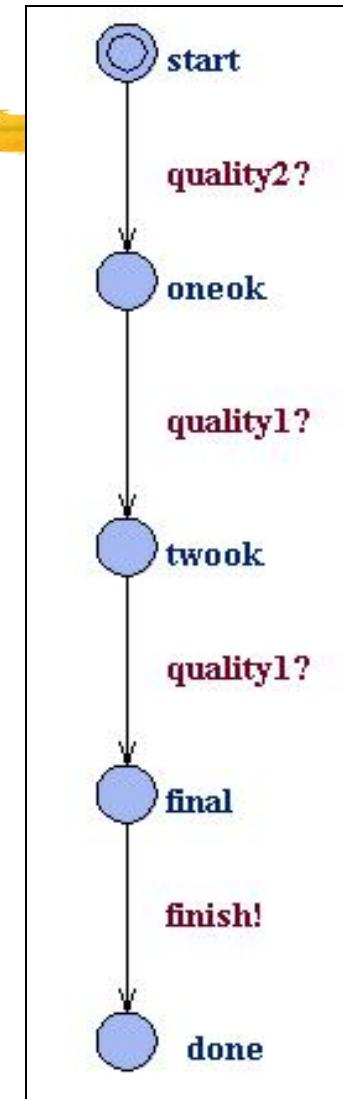
# Steel Recipe

- ⌚ Clock **tot** to constrain upper total time bound.
- ⌚ Clock **t** to measure duration of machine treatment.
- ⌚ Channel **quality1!** signaled when steel converted

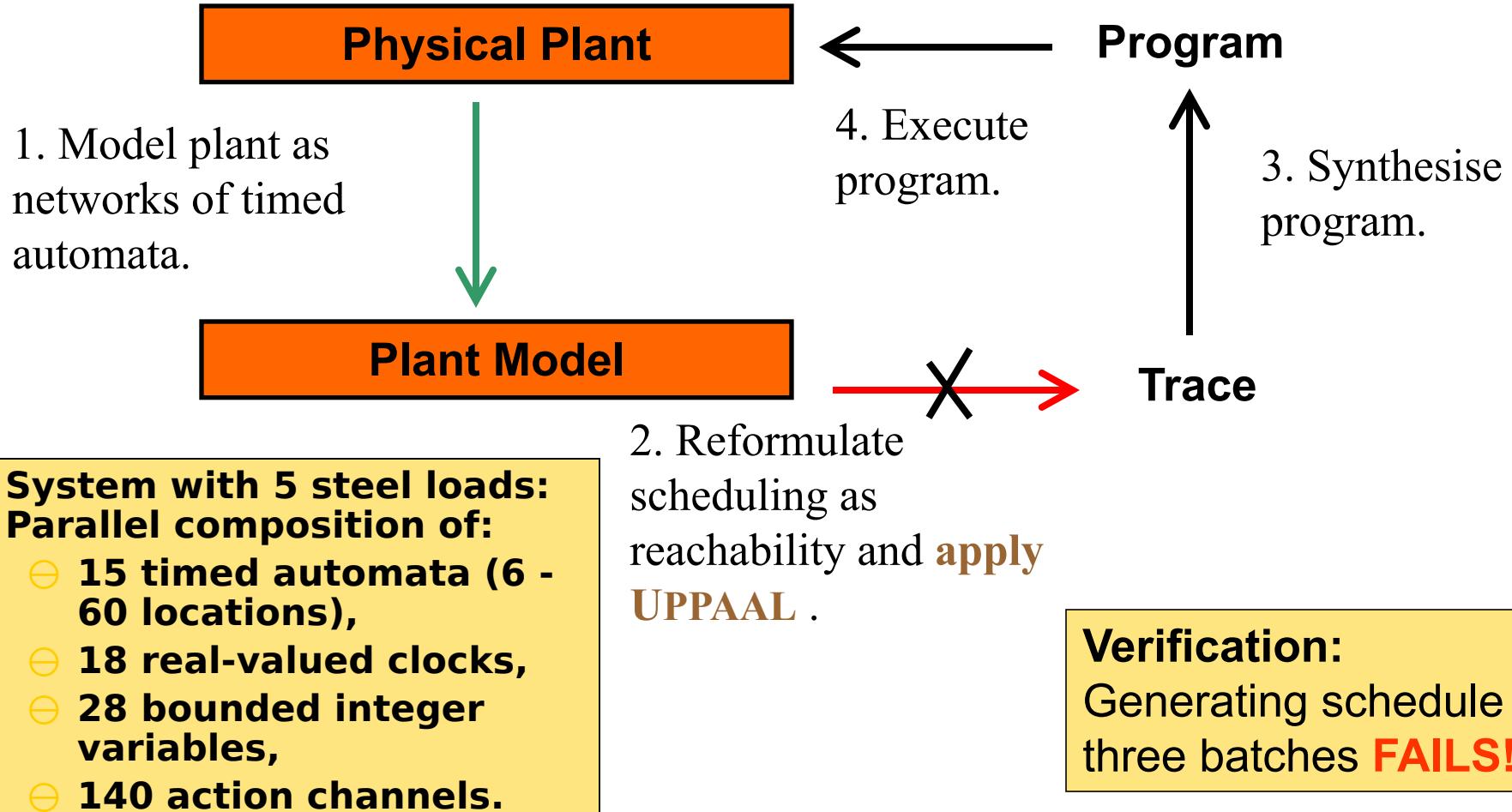


# Production Order

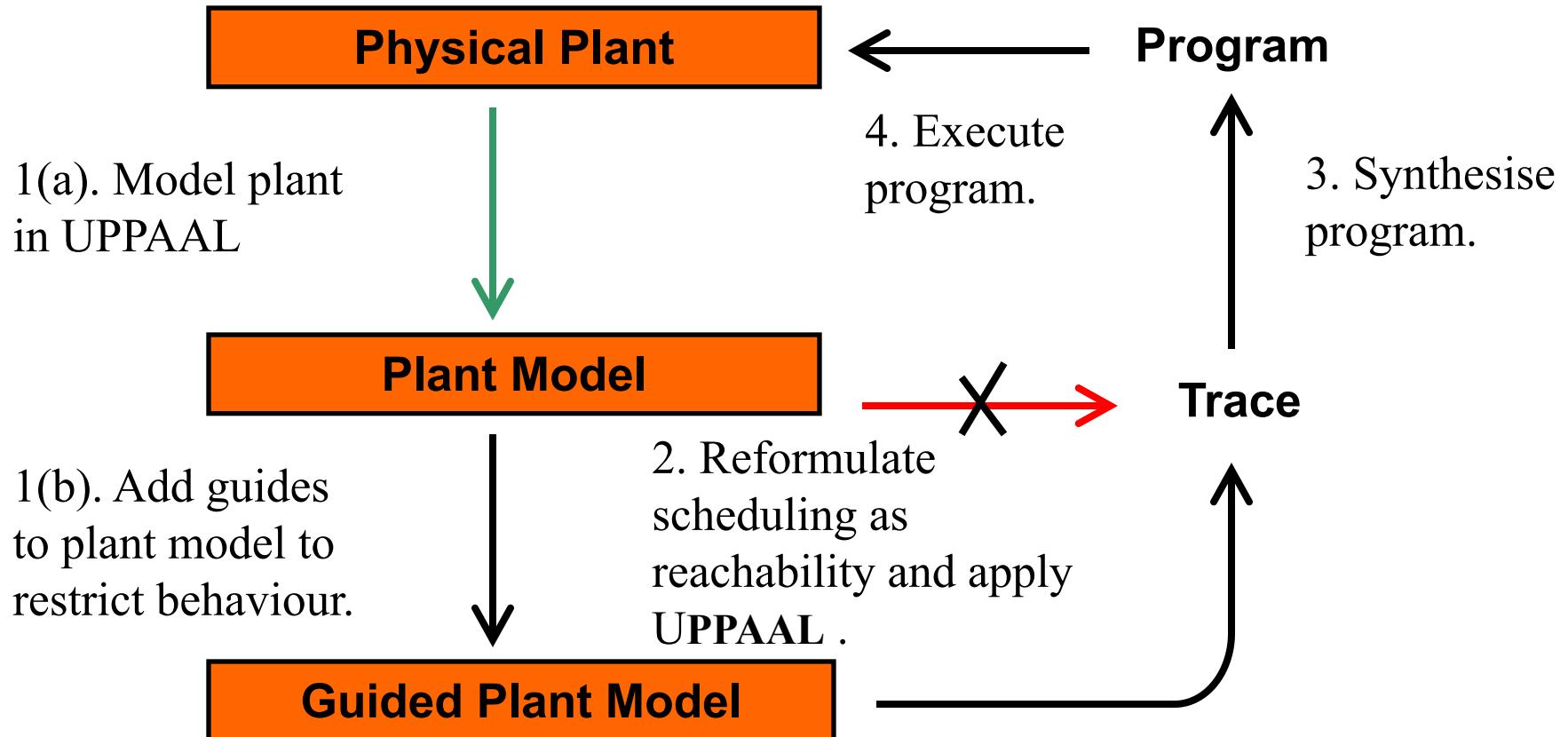
- ⌚ Specifies the steel qualities to be produced.
- ⌚ Allows for scheduling to be reformulated as reachability.
- ⌚ Reachable only with **feasible schedule**
- ⌚ E<>( final )



# Modus Operandi



# Modus Operandi



# Guiding the Model

**Idea:** Guide model according to chosen strategies:

- ⊖ **enforce** desired behaviors,
- ⊖ **restrict** undesired behaviors.

**Implementation:** Annotate model with:

- ⊖ clock and integer variables,
- ⊖ assignments (to added clock or variable), and
- ⊖ guards (to any clock or variable).

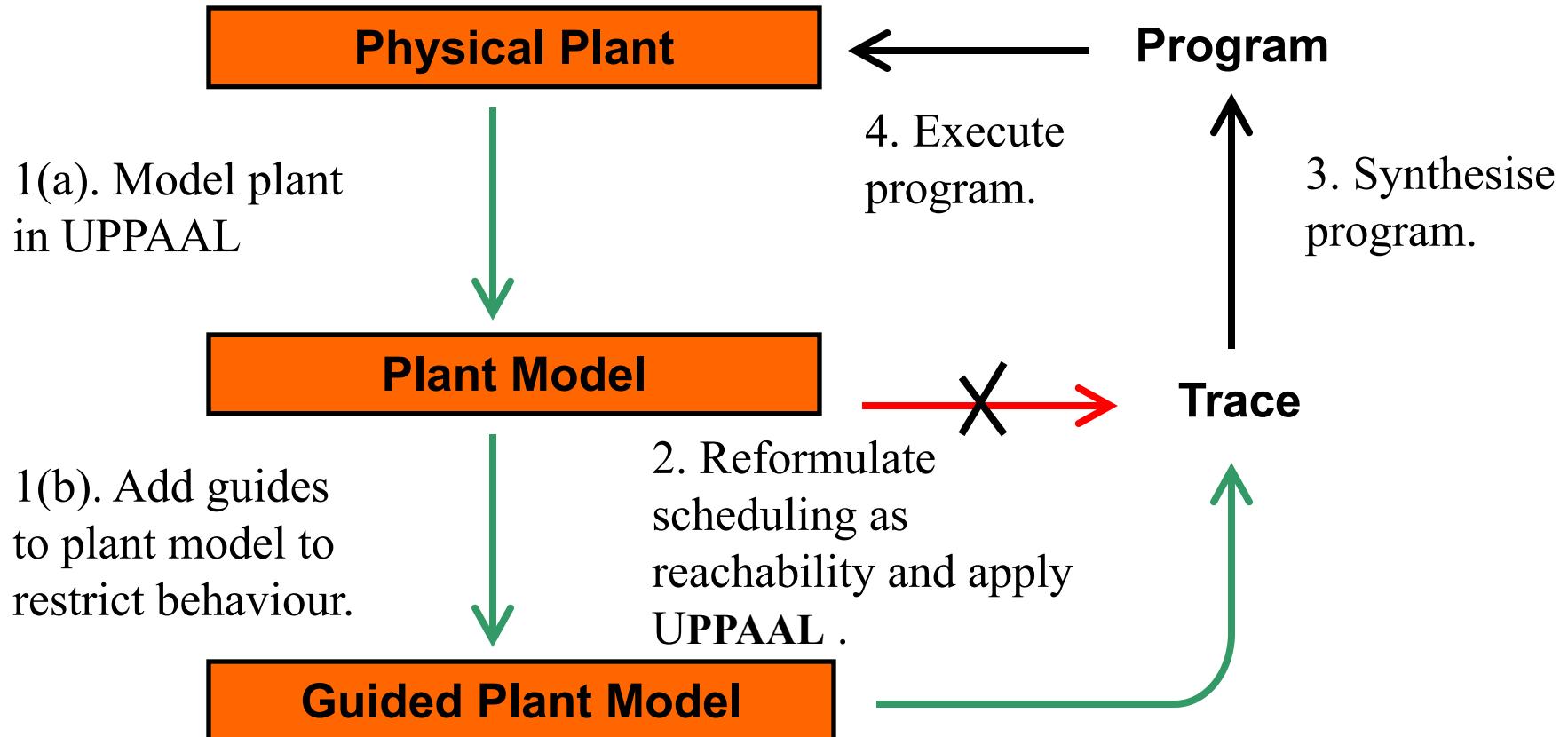
**Fact:** Trace of **guided** model is guaranteed to be trace of **unguided** model.

# Experiment

n	All Guides						Some Guides						No Guides					
	BFS		DFS		BSH		BFS		DFS		BSH		BFS		DFS		BSH	
	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB	s	MB
1	0,1	0,9	0,1	0,9	0,1	0,9	0,1	0,9	0,1	0,9	0,1	0,9	3,2	6,1	0,8	2,2	3,9	3,3
2	18,4	36,4	0,1	1	0,1	1,1	-	-	4,4	7,8	7,8	1,2	-	-	19,5	36,1	-	-
3	-	-	3,2	6,5	3,4	1,4	-	-	72,4	92,1	901	3,4	-	-	-	-	-	-
4	-	-	4	8,2	4,6	1,8	-	-	-	-	-	-	-	-	-	-	-	-
5	-	-	5	10,2	5,5	2,2	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	13,3	25,3	16,1	9,3	-	-	-	-	-	-	-	-	-	-	-	-
15	-	-	31,6	51,2	48,1	22,2	-	-	-	-	-	-	-	-	-	-	-	-
20	-	-	61,8	89,6	332	46,1	-	-	-	-	-	-	-	-	-	-	-	-
25	-	-	104	144	87,2	83,3	-	-	-	-	-	-	-	-	-	-	-	-
30	-	-	166	216	124,2	136	-	-	-	-	-	-	-	-	-	-	-	-
35			209	250	-	-	-	-	-	-	-	-	-	-	-	-	-	-

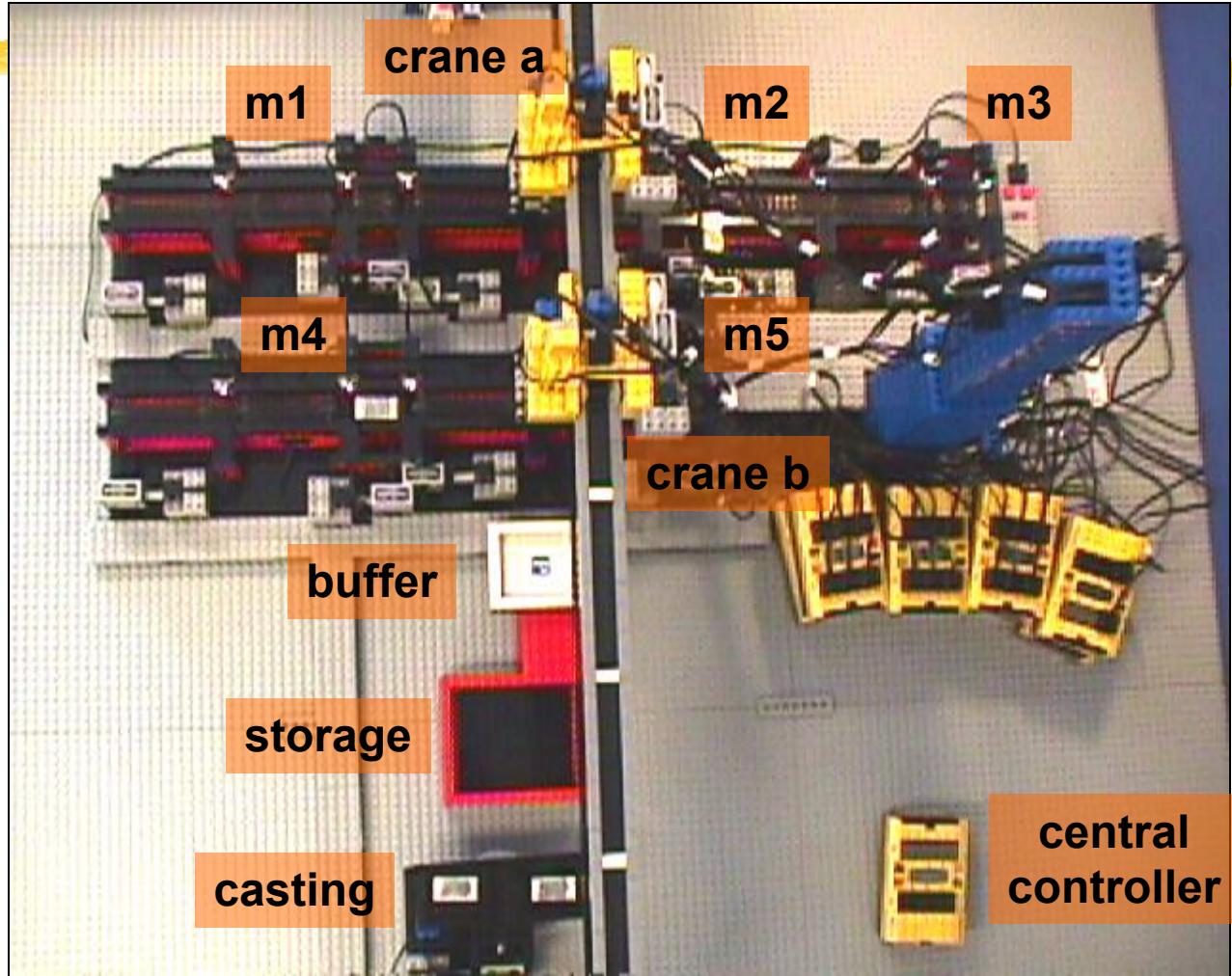
- **BFS** = breadth-first search, **DFS** = depth-first search, **BSH** = bit-state hashing,
- “-” = requires >2h (on 450MHz Pentium III), >256 MB, or suitable hash-table size was not found.
- System size:  $2n+5$  automata and  $3n+3$  clocks, if  $n=35$ : 75 automata and 108 clocks.
- Schedule generated for **n=60** on Sun Ultra with 2x300MHz with 1024MB in 2257s .

# Modus Operandi



# LEGO Plant Model

- ⌚ LEGO RCX Mindstorms.
- ⌚ Local controllers with control programs.
- ⌚ IR protocol for remote invocation of programs.
- ⌚ Central controller.



Synthesis

# Local Control Programs

## Belts:

- ⊖ move left/right,
- ⊖ receive from left/right,

## Machine:

- ⊖ start,
- ⊖ stop,
- ⊖ move left/right,
- ⊖ receive from left/right,

## Cranes:

- ⊖ move up/down,
- ⊖ set down,
- ⊖ pick up,

## Casting Machine:

- ⊖ start,
- ⊖ stop,

⊖ ...

# Extracting Programs

## Trace

...  
 ( loadB1.p1 recipeB1.gotoT1 loadB2...  
 { loadB1.x=5 recipeB1.tot=5  
 recipeB1...

### Sync: b1right

( loadB1.pre recipeB1.gotoT1 loadB2...  
 { loadB1.x=5 recipeB1.tot=5  
 recipeB1...

### delay( 5 )

( loadB1.pre recipeB1.gotoT1 loadB2...  
 { loadB1.x=10 recipeB1.tot=10  
 recipe...

### Sync: B1M1on

( loadB1.onM1 recipeB1.onT1 loadB2...  
 { loadB1.x=0 recipeB1.tot=10 recipe...

### delay( 10 )

( loadB1.onM1 recipeB1.onT1 loadB2...  
 { loadB1.x=10 recipeB1.tot=20  
 recipe...

### Sync: B1M1off

( loadB1.pre recipeB1.gotoT2 loadB2... Kim G. Larsen & Paul Pettersson

## Schedule

...

**belt1 right**

**delay 5**

**load B1 on Machine 1**

**delay 10**

**load B1 off Machine 1**

...

## Program

...

// Belt Unit 1 move  
 RIGHT  
**PB.SendPBMessage 2,**  
**20**

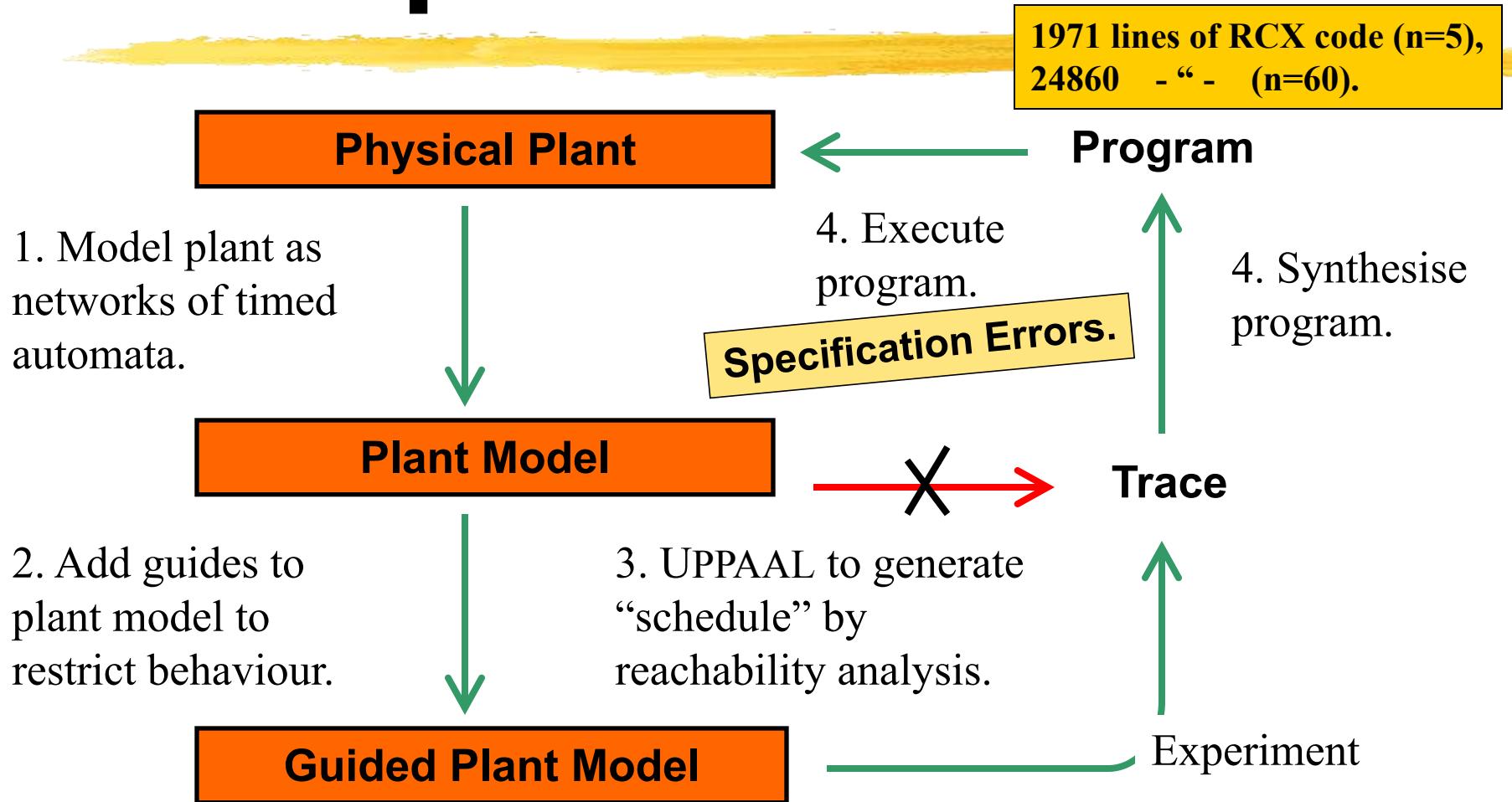
// Delay 5  
**PB.Wait 2, 500**

// Machine 1 START  
**PB.SendPBMessage 2,**  
**23**

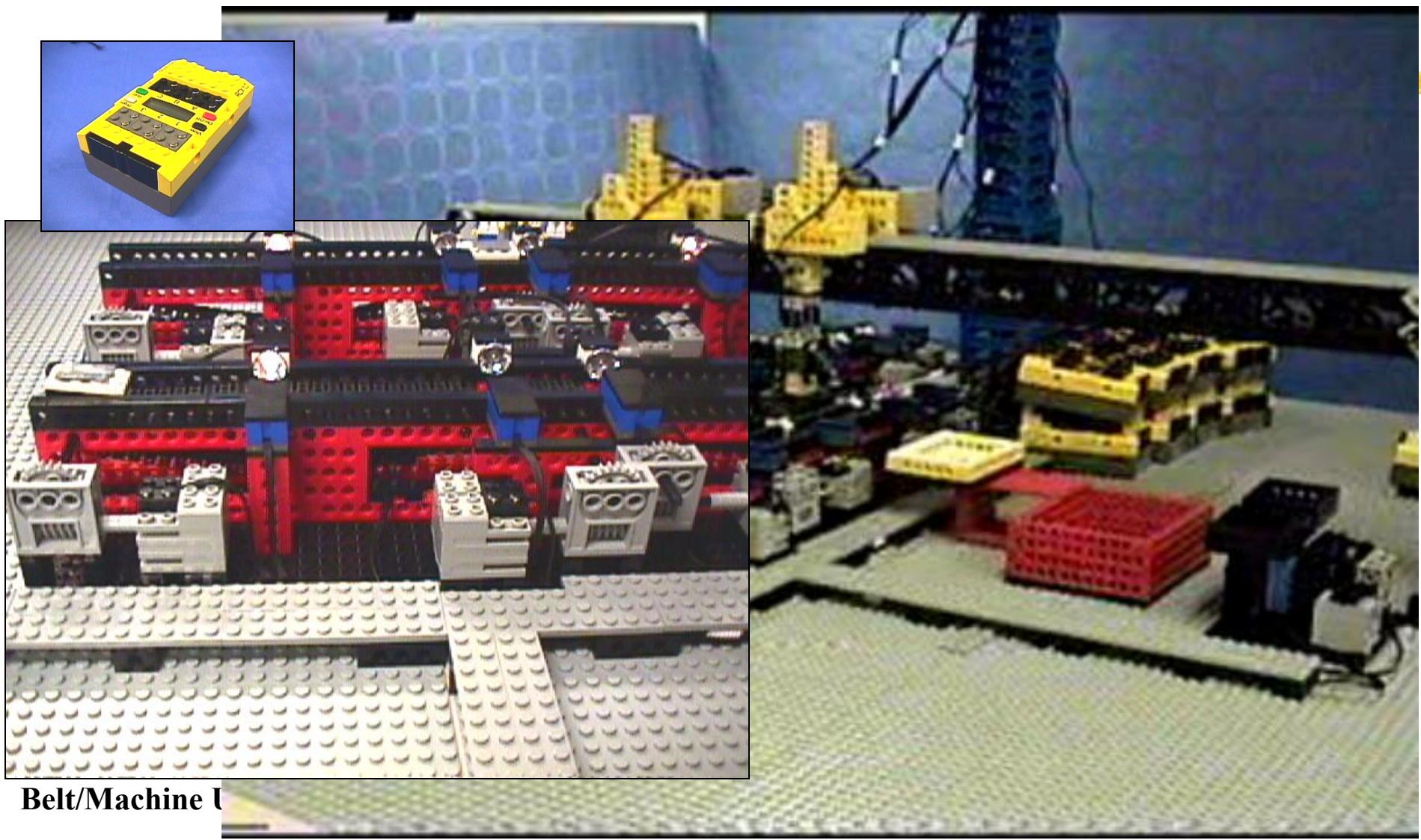
// Delay 10  
**PB.Wait 2, 100**

// Machine 2 STOP  
**PB.SendPBMessage 2,**  
**24**

# Modus Operandi



# LEGO Plant Model



# Current & Future Research

⌚ **DUPPAAL**

⌚ **PUPPAAL**

⌚ **GUPPAAL**

⌚ **CUPPAAL**

⌚ **HUPPAAL**

⌚ **PrUPPAAL**

# Current & Future Research

 **DUPPAAL**

CAV2k]

 **PUPPAAL**

 **GUPPAAL**

 **CUPPAAL**

**Distributed** [Behrmann, Hune, Vandraager,

**Parameterized**

**Guided**

**Cost-Optimal**

 **HUPPAAL**

**Hierarchical (UML)**

 **PrUPPAAL**

**Probabilistic**



**ENDE**